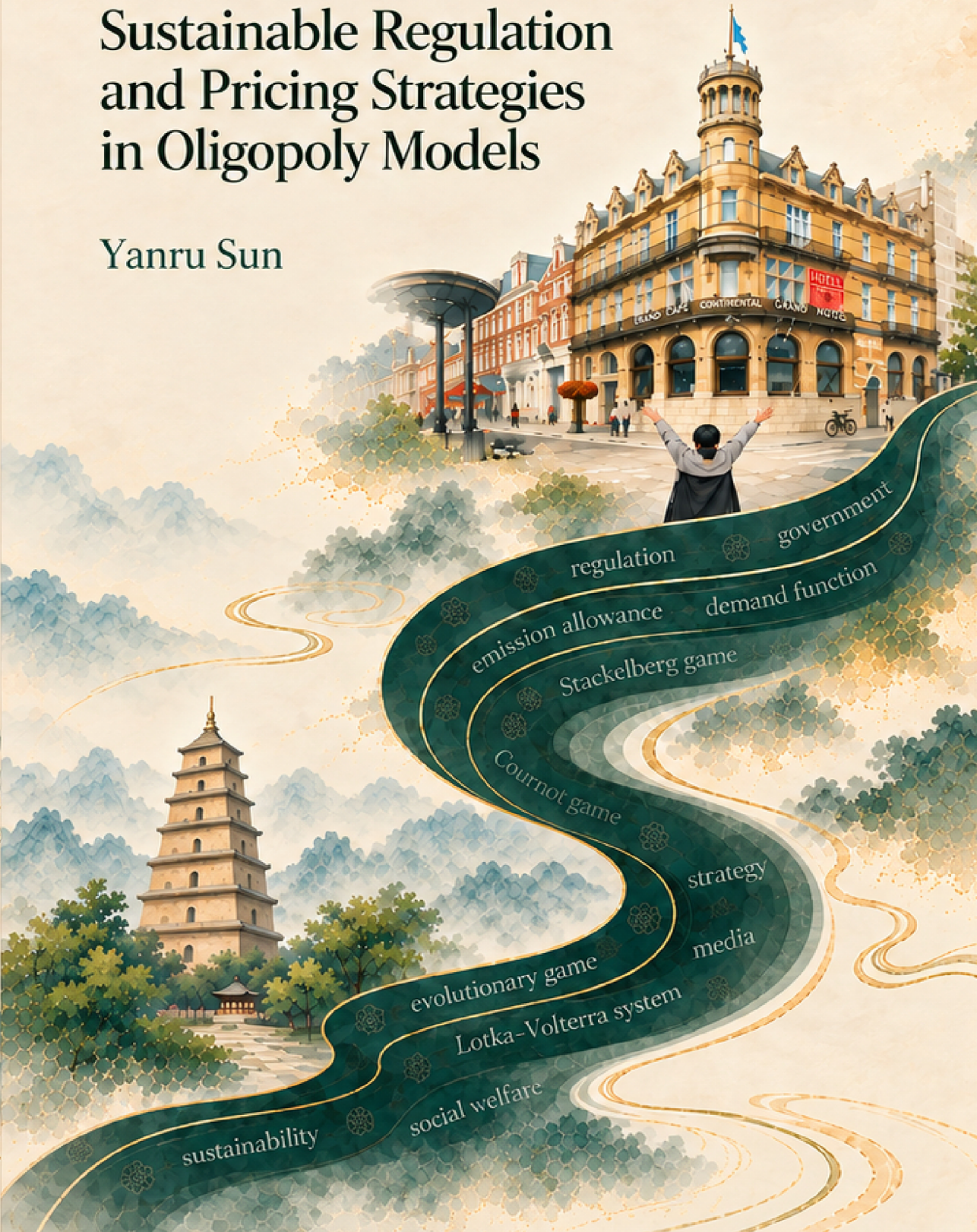
Sustainable Regulation and Pricing Strategies in Oligopoly Models

Yanru Sun



Propositions accompanying the PhD-thesis

Sustainable regulation and pricing strategies in oligopoly models

Yanru Sun

1. Media supervision can incentivize firms to adopt socially responsible strategies, even when the media is driven by profit maximization. (Chapter 2)
2. Firms choose strategies that will obtain maximum profits, whereas sustainable regulation encourages more sustainable production by providing external incentives. (Chapter 2 and 3)
3. In the long run, firms learn and adapt to evolving economic conditions and policy changes. (Chapter 2 and 4)
4. The standard procedures for allowance allocation do not necessarily lead to socially optimal outcomes, as they may reflect political considerations rather than social welfare maximization. (Chapter 3)
5. As the demand for products changes over time, forecasting sales and dynamically adjusting prices can help firms increase their profits. (Chapter 4)
6. A laissez-faire or stringent policy toward firms' emissions may hinge on a single minor shift in policymakers' attitudes.
7. Mathematical models cannot fully capture reality, but they can emphasize its key features by abstraction and thus provide insights.
8. The present generations have the responsibility to bequeath to future generations an Earth which will not one day prove to be irreversibly damaged by human activity.

- UNESCO, Article 4

Doctoral thesis

SUSTAINABLE REGULATION AND PRICING STRATEGIES IN OLIGOPOLY MODELS

Yanru Sun

2026

SUSTAINABLE REGULATION AND PRICING STRATEGIES IN OLIGOPOLY MODELS

Dissertation

To obtain the degree of Doctor at Maastricht University,
on the authority of the Rector Magnificus, Prof. Dr. Jan Smits and
the degree of Doctor of Philosophy at the Northwestern Polytechnical University,
on the authority of the President, Prof. Dr. Baowei Song,
in accordance with the decision of the Board of Deans,
to be defended in public
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1
Introduction

Economic activities such as production or task sharing are at the basis of society. However, many economic activities cause environmental damage. Examples run from pollution by chemical industries to agricultural degradation and emissions of greenhouse gases by coal-fired electricity generators. In the face of environmental threats, sustainability has received increasing attention, especially in emissions reduction, sustainable compliance strategies, and green technological innovation.

One strand of research studies these environmental issues and policy designs through oligopoly models. An oligopoly is a market in which a limited number of firms compete and in which the choices made by a firm affect not only the profits of that firm itself but also those of the other firms. Firms have to take the actions of other firms into account to maximize profits. In an oligopoly, the strategic choices of firms can be with regard to variables such as price or output, but also, for example, advertising, R & D investment, or quality.

Another aspect of economic activities is its dynamics over time. For

example, the demand for products is influenced by consumers' preferences, which may change over time. News reports may guide consumers to choose products with third-party certification, and with the awakening of green awareness, consumers gradually prefer sustainable compliance products over traditional ones.

As a foundational framework for such dynamic evolution, the replicator dynamic model describes how the aggregate behavior of firms using certain imitative protocols evolves over time. For example, the replicator dynamic equation determines the fraction of firms with different strategies in an evolutionary game, or the output of firms in a dynamic adjustment process.

In this thesis, we study these two aspects of economic activities, namely sustainable growth and the dynamic evolution of demand and production. Chapters 2 and 3 are devoted to sustainable growth, and Chapter 4 is devoted to the dynamic evolution of demand and production.

Chapter 2: A media supervision mechanism based on the Stackelberg evolutionary game model

Chapter 2 studies the impact of media supervision on the Corporate Social Responsibility behaviors of firms. Corporate Social Responsibility (CSR) means that business leaders have a responsibility to do more than simply maximize short-term profits, for example, also making contributions to sustainable development.

A CSR strategy is often more costly than a traditional strategy for manufacturers, leading to a large market share for traditional products. However, the pollution generated by the traditional strategy results in forest degradation, global warming, and other environmental damage. Therefore, sustainable production requires a transition from a traditional strategy to a CSR strategy.

Chapter 2 uses an evolutionary game to study the long-term competition between firms using a CSR strategy and firms using a traditional strategy. We consider an infinite time horizon and a continuum of

firms indexed by the unit interval. One fraction of firms uses a CSR strategy, while others use a traditional strategy. In each period, two firms are randomly selected from the infinite pool of firms. In that period, the selected firms play a Bertrand game. The Bertrand equilibrium in this game generates the payoff matrix for the evolutionary game, which governs the change in the fraction of firms that play the CSR strategy. We find that the CSR strategy is dominated by the traditional strategy. This implies that, absent intervention, firms using the CSR strategy will become extinct in the long run.

Therefore, external incentivization is necessary to increase the fraction of firms that use the CSR strategy. One way to achieve this is by media supervision. Empirical evidence, for example, Damberg et al. (2022), indicates that media supervision influences consumer demand through disclosing corporate behaviors, which ultimately affects the viability of firms choosing a CSR strategy.

We construct a Stackelberg evolutionary game to analyze the strategic interaction between the media and the manufacturer population. As a leader, the media decides on a level of supervision efforts, which affects demand and hence the profits of manufacturers. Manufacturers participate as followers in the evolutionary game under media supervision.

We show that there exists a unique Stackelberg evolutionary equilibrium. We show that, in the equilibrium with media supervision, there is a strictly positive fraction of firms that use the CSR strategy. An algorithm is designed to calculate the Stackelberg evolutionary equilibrium. Finally, we extend the model to a situation where consumers dynamically adjust their sensitivity to environmental issues. In this model, we find the optimal level of media supervision. Numerical simulations show that when green sensitivities change dynamically, there also exists a unique optimal effort level for the media, stabilizing the fraction of firms using the CSR strategy at a high value.

Chapter 3: Maximizing Social Welfare: Emission Allocation in a Cournot Model

Chapter 3 analyzes the equilibrium of the Cournot game under emission allowances allocated by the government, and explores the optimal allocation to maximize social welfare.

To control carbon emissions, many countries and governments have implemented the cap-and-trade system. Policymakers set a cap on emission by allocating emission allowances to firms. Firms can trade these emission allowances and subsequently produce under the constraint that carbon emissions should not exceed the total volume of allowances obtained. Such allocation mechanisms have been widely studied and are known to affect the outcome of emission reduction. For example, Cramton and Kerr (2002) found that the auction had a better effect on reducing emissions than grandfathering, which allocates emission allowances based on historical output.

However, few studies have explored the goals that the government aims to achieve. For instance, the government may focus on environmental welfare, economic welfare, or a balance between the two. This depends on the attitude of society that this government represents. Different attitudes towards environmental welfare may lead to different optimal choices regarding the allocation of emission allowances.

Given pollution emissions, Chapter 3 first constructs a Cournot model between two firms in the absence of regulation. This market consists of a "dirty" firm and a "clean" firm, in which the upper limit of the pollution level of the dirty firm is greater than that of the clean firm. Moreover, the more polluting the technology is, the cheaper production becomes. Under perfect information, each firm simultaneously selects a quantity and a pollution level to maximize its profit. We argue that the optimal strategy for each firm is to adopt the upper limit of the pollution level in the Cournot equilibrium.

The government is endowed with a social welfare function that takes social damage into account. Social damage includes, for example, pollution, health costs, acid rain, global warming, and other environmental harms. To maximize social welfare, the government can cap social damage by allocating emission allowances to firms.

We propose a Stackelberg game model to analyze the maximization of social welfare and compute the corresponding optimal allocation. In the first stage, the government decides on an allocation of emission allowances, which put a cap on the total volume of carbon emissions of each firm. In the second stage, firms play a Cournot game with capacity constraints due to emission allowances. The first stage is then solved using backwards induction.

In the Cournot equilibrium, the output and the pollution level are continuous as functions of the emission allowances. Therefore, in the first stage, the government can directly maximize social welfare through the allocation of emission allowances. In equilibrium, the allocation of allowances by the government is not a continuous function of social damage. Rather, depending on the social damage in the Cournot equilibrium, the government chooses one out of a limited number of allowance allocations. We also analyze a special case where the social damage is a linear function of total emissions. In this specific case, under mild conditions, the government-optimal strategy is either to enforce zero pollution or to allow firms to pollute freely.

Chapter 4: A Dynamic Surge Pricing Model throughout Product Lifecycle

The rise of e-commerce has led to rapid changes in product demand due to product diffusion, social networks, and advertising. Throughout the product lifecycle, the changes in demand include the rapid growth phase (the 'surge'), the slow growth phase, and the maturity phase (X. Li et al., 2021). To match real-time demand in each period, firms need dynamic pricing throughout the product lifecycle (Tao et al., 2021).

Motivated by the above issues, Chapter 4 uses a dynamic pricing model called the surge pricing model to forecast demand and price during the product lifecycle. It extends the classical Cournot model by introducing dynamic feedback between price and demand. This results in a dynamic pricing model that is in fact equivalent to the Lotka-Volterra system of differential equations. It is also equivalent

to the equations of the replicator dynamics, which is a model of biological competition among members of two species (Hofbauer and Sigmund, 1998).

We show the effectiveness of the dynamic surge pricing model. First, a definition of the product lifecycle curve is given to theoretically describe the sales trajectories of products. A lifecycle curve is a smooth, non-decreasing function that starts with zero demand and is constant after a certain time period. Then we prove that the solution function of the sale-forecast system is indeed a product lifecycle curve. Based on the sales forecast, the price adjustment extends the inverse demand function by incorporating the impact of marginal sales. The dynamic price is a smooth function which initially increases, then decreases, and ultimately stabilizes at a constant level.

We derive sufficient conditions for the global asymptotic stability of the equilibrium points of the dynamic surge pricing model. Finally, we discuss the parameter conditions for the dynamic surge pricing equilibrium to be stable at the Cournot equilibrium. Particularly, the price function degenerates into a linear inverse demand function in the Cournot model under certain conditions. We use a series of numerical experiments to analyze the value range and the influence on the price and profit of each parameter. The numerical results indicate that dynamic pricing performs better than static pricing, which complies with the data analysis by Tong et al. (2020).

2

A media supervision mechanism based on the Stackelberg evolutionary game model

This chapter is based on: Yanru Sun, Hao Sun, Panfei Sun, Xuanzhu Jin, and Yimei Yang (2025). "Elevating the corporate social responsibility level: A media supervision mechanism based on the Stackelberg-Evolutionary game model". In: *Omega* 131, p. 103215. DOI: https://doi.org/10.1016/j.omega.2024.103215.

Abstract

Environmental taxes alone may not solve the social dilemma posed by the conflict between the myopic pursuit of profit and the cost of corporate social responsibility (CSR). Designing a reasonable supervision mechanism is crucial to correcting market failures. We develop a media supervision mechanism through a Stackelberg evolutionary game model to study the impact of media supervision on the evolutionary behavior of the manufacturer population. Assuming the media is leader, manufacturers' demands are heterogeneous under different strategy profiles after the media determines the effort level of supervision. The best response of the manufacturer population is the evolutionarily stable strategy under supervision, where the percentage of CSR strategies is defined as CSR level. It is proved that the CSR level elevates with the increase of effort level. We analyze the existence and uniqueness of Stackelberg evolutionary equilibrium and a numerical algorithm to compute it. The results show that CSR level under Stackelberg evolutionary equilibrium is higher than that without supervision. Our research not only illustrates the effectiveness of media supervision in reducing environmental pollution but also provides suggestions for governments to formulate environmental policies and improve regulatory mechanisms.

2.1 Introduction

With the rapid economic development, the global community is under threat of global warming, air pollution, energy depletion and other environmental dilemmas as never before (Khosroshahi et al., 2021). In order to deal with these problems, moving towards green manufacturing is an inevitable trend, which needs manufacturers to undertake social responsibilities in production and operations (Ni and K. W. Li, 2012; Besiou and Van Wassenhove, 2016). For instance, the conventional solvent used in the printing industry is ink, whereas the methyl ester solvent developed by Arkori and NuPro Technologies exhibits low toxicity and degradability through corporate social responsibility (CSR) investments. Another motivating case for our research is found in the realm of automotive energy. Tesla's electric vehicles embrace clean energy alternatives in lieu of conventional fuels like diesel and gasoline (Zheng et al., 2022).

In these scenarios, the implementation of the CSR strategy inevitably leads to increased production costs for manufacturers. Consequently, rooted in manufacturers' intrinsic self-interest, supervision and punishment are necessary to correct the market imperfections (X. Ma et al., 2021). Recent research on environmental policy largely revolves around environmental taxation (Zhou et al., 2021), encompassing analysis of tax design and its subsequent impacts. Despite the effectiveness of environmental improvement, tax policies can also bring undesirable results in other aspects. For instance, with the strictest carbon tax in the world, British Columbia achieved emission cuts while hurting economic growth, shrinking its labor market advantage versus Canada by 50% in the five years after the implementation of the carbon tax (Institute, 2016).

The main reason for the problem is that it takes a long time to change consumption through environmental taxes, resulting in a disequilibrium in the market. Thus, it is necessary to timely upgrade the consumption structure through media supervision. Empirical evidence showed that media supervision influences consumer demand through

disclosing firms' behaviors, which ultimately affects the CSR level (Jun Li and Wu, 2020; Luo et al., 2022). For example, media coverage of the Volkswagen and Ford pollution incidents resulted in a negative impact on the demand for traditional energy vehicles, promoting the development of the environmentally friendly new energy vehicles (Awasthy et al., 2022). Another example comes from China where media bias is less due to the restricted media environment. A sample of 332 corporate violations from 1998 to 2018 shows that media plays a governance role at the firm level by changing consumer perceptions and demands (Z. Jiang et al., 2022).

Motivated by the above-mentioned problems, we study the media supervision of manufacturers in an industry with homogeneous competition, in which the cost of each strategy for manufacturers is essentially the same. The schematic of the interaction among government tax, media supervision and manufacturers' behaviors is shown in Fig. 2.1. In this chapter, the environmental tax imposed by the government is modelled as the external punishment on manufacturers and unilaterally influences the strategic choices of manufacturers. The game between the media and the manufacturer population is constructed as a Stackelberg evolutionary game model, in which the media is the leader deciding the effort level of supervision with quadratic cost. To maximize its profit, the media gets Web traffic and converts it into cash by disclosing the manufacturer's strategy. As followers are bounded rational, manufacturers play the evolutionary game with CSR or traditional strategy and have heterogeneous demand functions in different strategy profiles under media supervision. We define the CSR level as the percentage of manufacturers with CSR strategy in the evolutionarily stable state, which in turn has an impact on media profit.

Our main findings are as follows. First, we analyze the parametric sensitivity of each strategy profile without media supervision. In evolutionary equilibrium, this paper discusses the influence of environmental taxes, consumer greenness, and cost premiums on the CSR level, and the above findings are supported by a practical case of green manufacturing and traditional manufacturing.

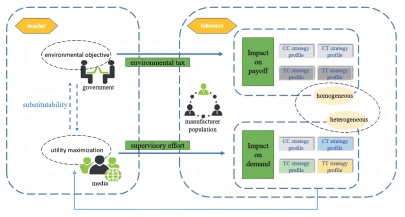


Figure 2.1: The illustration of the interaction between the government and firms.

Regarding the media supervision mechanism, the CSR level is shown to elevate with the increase of effort level. We demonstrate that corresponding to the profit-maximizing effort level for the media, there exists a unique response for manufacturers, resulting in a relatively high CSR level. This result shows the existence and uniqueness of the Stackelberg evolutionary equilibrium. Then we design an algorithm to calculate the Stackelberg evolutionary equilibrium and verify the above theoretical results through a set of numerical experiments. Finally, we extend the model to the dynamic situation to study the optimal effect of media supervision when consumers are constantly learning and updating their green sensitivity. Based on the Runge-Kutta Method, an algorithm for computing the game equilibrium is developed in the dynamic setting. The results show that when green sensitivities change dynamically, there also exists a unique optimal effort level for the media, stabilizing the CSR level at a high value.

The topic of corporate governance has been a significant focus of recent research. For all we know, most of the literature examined mandatory regulation and punishment, and few studies considered the game between the for-profit regulator and manufacturers. Combining literature with practice, we highlights the following contributions.

From the perspective of modeling:

- In order to develop the media supervision mechanism, we propose a novel model that nests the evolutionary game into the

Stackelberg game. This model utilizes regulation strategies of the media to influence the bounded rational behaviors of the manufacturer population and analyzes the role of media in promoting green manufacturing in the long run.

- We define the concept of CSR level to measure the social contribution of the manufacturer population and reveal the impact of individual decisions on the overall environment. This definition offers a new perspective for quantifying the greenness and pollution emission of the manufacturer population.

- Empirical evidence has validated the regulatory role of media on manufacturers, but few researchers have discussed the profit and the optimal strategy of media. Therefore, we explore the game model between the media and the manufacturer population, and propose the heterogeneous impact of media supervision on each strategy profile. In contrast to the case without media supervision, the equilibrium of the Stackelberg evolutionary game is demonstrated to increase the CSR level when maximizing the profits of both media and manufacturers.

Our work also has important managerial insights:

- Our research reveals that environmental taxes and media coverage influence the strategic choices of the manufacturer population by affecting their payoffs and demands, respectively. To maximize profits, bounded rational individuals should adopt a mixed strategy in the long-term competition, which is using CSR strategies with a high probability and using traditional strategies in a few cases. It offers guidance for manufacturers to balance social responsibility and economic efficiency in production and operation under supervision.

- The Stackelberg evolutionary game model broadens the research methods of informal supervision mechanisms. We find that media exposure can alleviate information asymmetry for consumers

and force manufacturers to take active measures on environmental protection. As a for-profit regulator with disclosure cost, the click-through rate of media coverage can be converted into advertising, government subsidies and other revenue. Therefore, we suggest an optimal exposure effort of the media and emphasize the role of media in corporate governance.

- The results show that the media supervision mechanism effectively addresses the social dilemma of environmental governance. This emphasizes the necessity of a combination of media supervision and environmental taxes for sustainable development. For policy makers, giving full play to the role of media in public opinion and strengthening media management are key to preventing government failure caused by excessive intervention.

These insights will help policy makers and for-profit regulators respond to corporate violations of social and environmental standards under Sustainable Development Goals.

The related work is reviewed in Section 2.2. Section 2.3 analyzes the evolutionary game among manufacturers with environmental taxes. In Section 2.4, we propose the media supervision mechanism by a Stackelberg evolutionary game. Section 2.5 discusses the robustness of key parameters and extends the model to incorporate dynamic green sensitivities. Section 2.6 contains a discussion and conclusion. All proofs are collected in the Appendix.

2.2 Literature review

2.2.1 Incentive policies for CSR in sustainable operations

Incentivizing corporation social responsibility (CSR) through environmental policies has become a popular topic due to the importance of CSR in sustainable operations (Feng et al., 2022; Hsueh, 2015).

First, much attention has been paid on how CSR behaviors are affected by the internal incentives between manufacturers and retailers in the green supply chain. For example, Ni, K. W. Li, and Tang (2010) consider the allocation of CSR costs in a two-echelon supply chain under wholesale price contract and find that there is an inherent conflict between economy and CSR performance. Hsueh (2014) proposes a revenue sharing contract embedding CSR to achieve channel coordination, which can simultaneously improve CSR performance and each partner's profits under certain parameter settings. Under information symmetry and information asymmetry, P. Ma et al. (2017) propose a two-part tariff contract for the retailer to incentivize the manufacturer to improve CSR efforts. The results show that the CSR cost has different impacts on the profits of the manufacturer and the retailer. Focusing on the liability costs caused by quality defection, Fan et al. (2020) develop a QD-QICS contract to effectively coordinate decentralized supply chains regardless of the channel leadership structures. Some studies also analyze CSR behaviors in the supply chain through other contracts, such as cost sharing contract (Ghosh and Shah, 2015), innovation effort requirement contract (P. Zhang et al., 2021), and cut-off mechanism (Raj et al., 2021).

On the other hand, most of the extant studies focus on the external incentives of manufacturer competition, especially mandatory regulation policies such as environmental taxes. Considering the firms' goal of profit maximization, Krass et al. (2013) examine operational decisions in the face of environmental tax regulations. They find that an extremely high tax rate can have a negative impact on firms' green behaviors. Z. Li et al. (2023) analyze the impact of carbon tax on manufacturers' product line strategies with valuation uncertainty and information disclosure. Bárcena-Ruiz and Sagasta (2021) compare the incentive effects of environmental taxes, cap-and-trade policies and emission standards on CSR level and social welfare when polluting firms care about CSR. Combining CSR and the market structure, Hafezalkotob et al. (2023) investigate the influence of Direct Tariff on Market and other four environmental policies on the sustainability performance of

competing smart supply chains. Besides, environmental taxes, subsidies and cap-and-trade policy are common policies. Existing literature indicates that government subsidies also play an important role in CSR incentives (R. Chen et al., 2023; Arya and Mittendorf, 2015; W. Huang et al., 2019). Cap-and-trade is regarded as a palatable alternative to a carbon tax, in which the government caps carbon dioxide emissions of each company and allows companies to sell or trade unused credits (Anand and Giraud-Carrier, 2020; Marufuzzaman et al., 2014; Bai et al., 2023; Mondal and B. C. Giri, 2022).

Most of the above literature consider the incentives of regulators to manufacturers but do not analyze the profits of regulators. In contrast, this chapter takes media as an example to study the game between a for-profit regulator and manufacturers, thereby analyzing the regulatory effects. In addition, most of the literature focuses on the CSR strategies of a few firms and ignores the overall CSR evaluation in the multi-manufacturer market. Therefore, a definition of CSR level is proposed to measure and enhance CSR performance in an industry of homogeneous competition in this paper.

2.2.2 Media supervision

The existing empirical research on environmental regulation suggests that as an informal external supervision mechanism, the media plays an important role in corporate governance (C.-C. Teng and J. J. Yang, 2021; X. Ma et al., 2021; Zyglidopoulos et al., 2012). Through analyzing the scenario of firms with news coverage about climate change in the Wall Street Journal, Hsu and T. Wang (2013) study the impact of the media exposure of CSR activities on market reactions to show that media regulation reduces environmental violations of firms. Damberg et al. (2022) make an empirical analysis of more than 700 US-based corporations from 2002 to 2014 and find that companies significantly increase the social and environmental responsibility after being the subject of negative news. Using Twitter's data as an example, Srinivasan et al. (2023) investigate the impact of social media exposure on the firm

value during large-scale disruptions, such as the COVID-19 pandemic. They confirm that the dissemination of information about CSR activities by social media contributes to preserving firm value. Kouwenberg and Phunnarungsi (2013) find that listed firms in Thailand face different market reactions according to their private reputation when their non-CSR-compliant operations are disclosed by the media. But in general, media regulation promotes CSR performance regardless of corporate reputation. Jinlong Li and Xiaoguang Xu (2024) test data from Chinese listed companies between 2010 and 2020, showing that media reports on CSR can influence public judgment to achieve an external governance effect. Utilizing a four-party evolutionary game model, Pan et al. (2023) indicate that media attention can motivate key participants to fulfill their environmental and social responsibilities by exerting public opinion pressure on them.

Most of the literature only studies the crucial role of media in corporate governance through a large number of empirical evidence and econometric models, but few theoretical models have been proposed to analyze media regulation for sustainable operations. All of them neglect the cost structure and profit mode of media, as well as the response of manufacturers to pursue the expected profit maximization under media regulation. To fill this gap, we model the strategy and profit function of the media and the heterogeneous effects of media supervision on manufacturers' demand, thereby exploring the regulatory motivation of media and the CSR level of the manufacturer population in equilibrium.

2.2.3 Game analysis in sustainable operations

The last literature stream related to this paper is the application of game theory to the regulation of sustainable operations. Some studies assume that all participants are bounded rational (Hosseini-Motlagh et al., 2022; Lahkar and Ramani, 2022; Johari et al., 2019). Taking the central regulatory strategy into account, Shi et al. (2022) construct a dynamic punishment mechanism to reduce enterprise pollution through

evolutionary game theory. A multi-party evolutionary game is developed to explore the sustainable implementation of carbon-labeling policy under governmental supervision (K. Jiang et al., 2023). W. Chen and Hu (2018) use evolutionary game theory to study the behavioral strategies of firms under various carbon taxes and subsidies. Using the evolutionary game on the complex network, Han et al. (2022) study the evolutionary behavior of research & development (R&D) under government subsidies in the new energy vehicle industry. The results show that both subsidies and dual-credit policies can increase the proportion of R&D strategy in the manufacturer population. Silva Rocha and Salomão (2019) construct an evolutionary game model with well-mixed and structured populations to discuss the interplay between corporate environmental compliance and policy regulation when facing a pollution trap.

Some existing studies assume that the players are rational (Adida and DeMiguel, 2011). Based on the framework of the Stackelberg game model, R. N. Giri et al. (2019) study government intervention on a green supply chain with two manufacturers and a retailer. Considering collaborative innovation by manufacturers, Hafezi et al. (2023) use a game-theoretic approach to study the pros and cons of cost sharing and innovation sharing in horizontal competition. Orsdemir et al. (2019) demonstrate that vertical integration and horizontal sourcing ensure corporate social and environmental responsibility and show that demand externalities have effects on CSR behaviors in the game equilibrium. Bian et al. (2021) investigate the impact of exposure risk on the demand of CSR or non-CSR compliant operations under Cournot competition. The results show that choosing CSR compliant operations is a win-win equilibrium if the external risk is high or if the cost premium is low. By a game-theoretical model with two firms, Zhu et al. (2023) propose incentives for CSR operations and how these depend on exposure probability, demand sensitivity and the cost premium.

However, the participants in game are either all regarded as rational or all characterized as bounded rational in the above works. But in practice, the for-profit regulator is rational and dominates in the game,

making the optimal strategic choice first. And manufacturers are limitedly rational, constantly learning and adjusting strategies in the long run. Thus, we develop a Stackelberg game embedded with an evolutionary game to explore the influence of media supervision on the evolutionary behavior of the manufacturer population.

Based on the review of the above literature, the studies most closely related to the our basic model are Bian et al. (2021) and Zhu et al. (2023). Both articles investigate the impact of demand externalities on CSR performance through game models. However, several different settings of our model which bring novel insights are listed below. First, our research proposes the heterogeneous impact of media supervision on manufacturers' demand, which is not considered in the prior works. Second, whereas they ignore the strategy and profit of regulators, we treat the media as a player with the goal of profit maximization. Finally, we develop a new Stackelberg evolutionary game model between a rational leader and bounded rational followers.

2.3 Model preliminaries

We focus on industries with homogeneous competition, in which many manufacturers produce goods at essentially the same cost and profit, such as bottled water, agricultural products, precious metals and paper industry. For instance, as the world's largest paper-producing nation, China has 7,288 paper manufacturers and the production cost is stable at 4.5 yuan per kilogram in 2022 (Statista, 2024). As another example, consumers typically pay $ 0.06 for an ounce of mineral water representing such brands as Smartwater, Volvic, Essentia, Perfect Hydration, etc.[1] Under the trend of green consumption, sustainable Packaging with additional costs is developed and used by brands of Green sheep water, Dasani and others.

[1]The data of price comes from Amazon.

In this context of many symmetrical manufacturers, we first consider the situation where manufacturers are charged environmental taxes but are not subject to media supervision and exposure. Due to the bounded rationality, the manufacturer population achieves optimal results in dynamic evolution through learning and imitating the behavior of others. In contrast to the static game model, the dynamic evolutionary game of population is related to population states (or strategy distributions). Therefore, we define the evolutionary game of the manufacturer population without media supervision as follows.

Definition 2.1. *An evolutionary game of the manufacturer population without media supervision is defined as* $G^w =< N, S, X, \Pi^w >$*, where*

(a) $N = [0, 1]$ *is the population of manufacturers;*

(b) $S = \{C, T\}$ *is the set of strategies;*

(c) $X = \{(x_C, x_T) \in \mathbb{R}^2_+ : x_C + x_T = 1, 0 \leq x_C \leq 1\}$ *is the set of population states;*

(d) $\Pi^w : S \times X \to \mathbb{R}^2_+$ *is the expected payoff function.*

The CSR strategy is $C \in S$ and the traditional strategy is $T \in S$. Further, $x_C \geq 0$ represents the proportion of the population using strategy C, and $x_T \geq 0$ represents the proportion of the population using strategy T. $\Pi^w = (\Pi^w_C, \Pi^w_T)$ denotes the expected payoff of each strategy in S.

In the evolutionary game, two firms are randomly selected from the population in each time step [2]. Thus, in the fixed state (x_C, x_T), the expected payoff is the average payoff that a firm (with C or T) obtains when matched with a firm using any possible strategy. Let Π^w_{CC}, Π^w_{CT},

[2]This model is identical to Deltas and Engelbrecht-Wiggans (2005), in which agents are randomly selected from a population of naive and rational bidders, and the fraction of each bidder type evolves based on their relative payoffs in the long run. This model fits well with the situation where firms compete, for example, for contracts repeatedly over time.

Π_{TC}^w and Π_{TT}^w represent the payoffs in all possible strategy profiles. Then the expected payoffs of each strategy under (x_C, x_T) are:

$$\Pi_C^w = x_C \cdot \Pi_{CC}^w + x_T \cdot \Pi_{CT}^w, \tag{2.1}$$

$$\Pi_T^w = x_C \cdot \Pi_{TC}^w + x_T \cdot \Pi_{TT}^w. \tag{2.2}$$

The average payoff for the population system is given by:

$$\Pi^w = x_C \cdot \Pi_C^w + x_T \cdot \Pi_T^w. \tag{2.3}$$

The payoffs Π_{CC}^w, Π_{CT}^w, Π_{TC}^w and Π_{TT}^w are calculated from a duopoly game. Then we give the duopoly model and solve for the equilibrium.

2.3.1 The payoff matrix for the evolutionary game

After two manufacturers have been randomly selected, we discuss the profits of the duopoly model in different strategy profiles. Due to symmetry, the strategy profile has three different structures, which are TT, TC/CT, CC. In the absence of media exposure, we assume that some consumers are active in environmental protection such that they will pay attention to whether the product is green in a personal way. Assuming that q_{CS_j} and q_{TS_j} denote the demand of strategy S_i against strategy S_j, $S_i, S_j \in \{C, T\}$, then the demand functions are given as follows,

$$\begin{cases} q_{CS_j} = \alpha - \beta p_C + \gamma p_T + \mu_0, \\ q_{TS_j} = \alpha - \beta p_T + \gamma p_C - \mu_0, \end{cases} \tag{2.4}$$

where p_C and p_T mean the price of each strategy. α is the market potential aggregate demand and μ_0 means the demand sensitivity of active

consumers to manufacturers' strategies. β and γ measure the self-price sensitivity and cross-price sensitivity of demand, respectively. In what follows, we derive the results of each strategy profile by calculating the optimal pricing and payoffs.

First, we examine the strategy profile CC with an encounter between two of the same strategy C. The CSR strategy creates cost premiums in business management, such as wage increases, innovation investment, and purchasing green materials. We assume that the additional cost of per unit product Δc that satisfies $\Delta c > 0$. And the lower the additional cost, the better the green technology. To maximize profit, both firms should solve the following optimization problems:

$$\begin{cases} \max\limits_{p_C} \Pi_{CC} = (p_C - c - \Delta c) q_{CC}, \\ \max\limits_{p_C} \Pi_{CC} = (p_C - c - \Delta c) q_{CC}. \end{cases} \tag{2.5}$$

In strategy profile CT/TC, one manufacturer takes C strategy, while the other uses T strategy. Without loss of generality, the firm with strategy C and the firm with strategy T have the following objectives:

$$\begin{cases} \max\limits_{p_C} \Pi_{CT} = (p_C - c - \Delta c) q_{CT}, \\ \max\limits_{p_T} \Pi_{TC} = (p_T - c) q_{TC}. \end{cases} \tag{2.6}$$

Strategy profile TT considers that both manufacturers follow strategy T to avoid cost premiums. Under this structure, the profit functions are changed to:

$$\begin{cases} \max\limits_{p_T} \Pi_{TT} = (p_T - c) q_{TT}, \\ \max\limits_{p_T} \Pi_{TT} = (p_T - c) q_{TT}. \end{cases} \tag{2.7}$$

To simplify the notation, we use $\Pi_{S_i S_j}$ to represent the payoff of a firm with S_i against S_j in the equilibrium. After solving the above opti-

mization problems, the value of $\Pi_{S_iS_j}$ ($S_i, S_j \in \{C, T\}$) and derivations are summarized in Section 2.7.

Then we construct the payoff table of each strategy profile with the environmental taxes on the basis of the above results. In this paper, the environmental taxes imposed by government is modeled as an external punishment for manufacturers. Based on the "polluter pays" principle, we assume that the government charges a constant environmental tax on each unit of payoff, and imposes an additional environmental tax on manufacturers using the T strategy to encourage manufacturers' eco-efficient activities. The constant environmental tax can be regarded as a proportional tax, where $\theta \in [0, 1]$ represents the basic tax rate and $\Delta\theta \in [0, 1 - \theta]$ measures the additional constant environmental tax to strategy T.

With the (additional) constant environmental tax, Table 2.1 gives the payoff of each manufacturer ($\Pi^w_{S_iS_j}$), where the detail of each value can be found in Section 2.7.

Table 2.1: The payoff table with environment taxes

	Strategy T	Strategy C
Strategy T	$\Pi^w_{TT} = (1 - \theta - \Delta\theta)\Pi_{TT}$	$\Pi^w_{TC} = (1 - \theta - \Delta\theta)\Pi_{TC}$
Strategy C	$\Pi^w_{CT} = (1 - \theta)\Pi_{CT}$	$\Pi^w_{CC} = (1 - \theta)\Pi_{CC}$

Based on the payoff matrix, we use the evolutionary game to evaluate the behavior of the manufacturer population that are bounded rational.

2.3.2 The evolutionarily stable strategy

According to the comparison of manufacturers' individual payoff with the population's expected benefits, manufacturers in the population imitate and learn from the behaviors of the others in the long run until the proportion of each strategy reaches a stable level, which is defined as evolutionarily stable strategy:

Definition 2.2. *A strategy σ^* is called an evolutionarily stable strategy (ESS) if and only if one of the following conditions holds,*

(a) $\Pi(\sigma^*, \sigma^*) > \Pi(\sigma, \sigma^*),\ \forall\, \sigma \neq \sigma^*$,

(b) $\Pi(\sigma^*, \sigma^*) = \Pi(\sigma, \sigma^*)$ *and* $\Pi(\sigma^*, \sigma) > \Pi(\sigma, \sigma), \forall \sigma \neq \sigma^*$.

The evolutionarily stable strategy is a "mass action interpretation" of the Nash equilibrium by placing the game in a demographic context. In other words, if the population is in equilibrium, then the best thing for everyone is to follow the crowd and use strategy σ^*.

Then we use a specific type of evolutionary dynamics, called the replicative dynamic equation (Tadelis, 2013), to find the evolutionarily stable strategy. The principle is that the proportion of individuals using a strategy increases (decreases) if its payoff is bigger (smaller) than the average payoff in the population to achieve a payoff above the system average.

The replicator dynamics equation describing the evolution of percentages of each strategy over time is modelled as follows,

$$\begin{aligned} \frac{\mathrm{d}x_C}{\mathrm{d}t} &= x_C(\Pi_C^w - \Pi^w) \\ &= x_C \cdot x_T[(\Pi_{CC}^w + \Pi_{TT}^w - \Pi_{CT}^w - \Pi_{TC}^w)x_C + \Pi_{CT}^w - \Pi_{TT}^w]. \end{aligned} \tag{2.8}$$

Let the replicator dynamic equation (2.8) be $\mathrm{d}x_C/\mathrm{d}t = 0$. For notational simplicity, let x^* denote the percentage of firms who follow strategy C under the ESS. It is easy to know the percentage of firms with strategy T is $1 - x^*$. According to Eq. $(2.1) - (2.8)$, we can get the equilibrium points, which are ESS for the evolutionary game. The following propositions give conditions of the ESS:

Proposition 2.3. *Supposing that* $(\Pi_{CC}^w - \Pi_{TC}^w)(\Pi_{TT}^w - \Pi_{CT}^w) \neq 0$, *the ESS of evolutionary game* (2.8) *under payoff table 2.1 has the percentage* x^* *in the*

following case:

$$\begin{cases} x^* = 0 & \text{if } \Pi^w_{CC} < \Pi^w_{TC} \text{ and } \Pi^w_{TT} > \Pi^w_{CT}, \\ x^* = 1 & \text{if } \Pi^w_{CC} > \Pi^w_{TC} \text{ and } \Pi^w_{TT} < \Pi^w_{CT}, \\ x^* = 0, 1 & \text{if } \Pi^w_{CC} > \Pi^w_{TC} \text{ and } \Pi^w_{TT} > \Pi^w_{CT}, \\ x^* = \frac{\Pi^w_{TT} - \Pi^w_{CT}}{\Pi^w_{CC} + \Pi^w_{TT} - \Pi^w_{CT} - \Pi^w_{TC}} & \text{if } \Pi^w_{CC} < \Pi^w_{TC} \text{ and } \Pi^w_{TT} < \Pi^w_{CT}. \end{cases} \tag{2.9}$$

Since the replicator dynamics equation is one dimensional, the dynamics always converges to the ESS.

According to the percentage x^* of firms with strategy C under the ESS, a general conception is mathematically defined to characterize the green degree of the firms' population participating in an evolutionary game.

Definition 2.4. *For an evolutionary game G^w, CSR level refers to the percentage x^* of firms that use strategy C in the whole population under the evolutionarily stable state.*

As mentioned above, CSR means that a firm contributes to the environment, while traditional behavior pursues profit ignoring green environmental protection. Therefore, Definition 2.4 uses the percentage of manufacturers who employ strategy C to measure the greenness of an industry. From Proposition 2.3 we can know that the CSR level increases until it reaches the stable value x^*, which is consistent with the practical case. After collecting more than 1,800 reports, GoldenBee found that the quality of China's CSR reports continued to develop until it reached a stable state in the "13th Five-Year Plan" period, with the comprehensive index remaining at the level of 1,300 points (GoldenBee, 2020).

The influence of environmental tax $\Delta\theta$ on the CSR level is studied from Proposition 2.5.

Proposition 2.5. *Suppose that* $\alpha > max\{(c+\Delta c)(\beta-\gamma)-\mu_0, c(\beta-\gamma)+\mu_0\}$ *and* $\alpha(2\beta+\gamma) > max\{(2\beta^2-\gamma^2)(c+\Delta c)-(2\beta-\gamma)\mu_0-\beta\gamma c, (2\beta^2-\gamma^2)c+(2\beta-\gamma)\mu_0-\beta\gamma(c+\Delta c)\}$, *the CSR level under ESS* (2.9) *is nondecreasing with respect to the environmental tax* $\Delta\theta$.

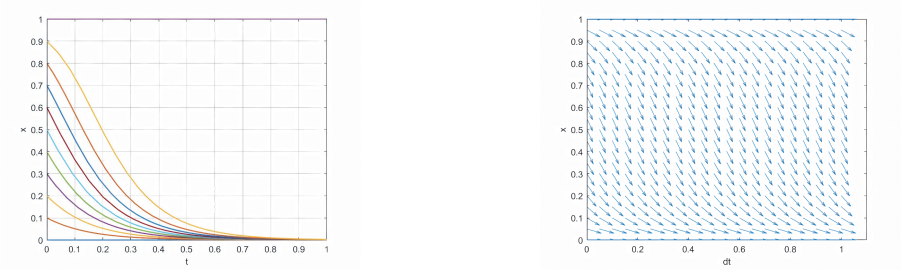
This parameter condition is a general economic constraint, which guarantees that the demand is always positive. Proposition 2.5 indicates that within a certain parameter range, the environmental tax charged by the government can reduce environmental pollution and encourage manufacturers to assume social responsibility. For example, the introduction of a carbon tax in Sweden since the 1990s has led to an increase in the CSR level. Measured by carbon emissions, it falls by more than 20% when the carbon tax was raised from €33 to €120 per tonne (Help, 2020). In addition, the Curbelo tax reduces environmental pollution in the United States by an additional 10 percent from the current downward trajectory (McGillis, 2019).

However, as an additional penalty, the environmental tax also has limitations in spite of improving the CSR level. Next, we explore the evolutionary characteristics and limitations through numerical analysis and practical cases in Subsection 2.3.3.

2.3.3 Numerical simulations

With the development of the economy and the deterioration of the environment, the competition between green goods and traditional goods is growing. According to Accenture's survey (Cai et al., 2022), green goods are more popular with consumers but carry extra costs for manufacturers. For example, Intel's 2021-22 Corporate Responsibility Report showed that product upgrades have generated $79 billion in revenue through investment. Compared to 2019, Intel reduced emissions by about 19% on an absolute basis, and saved 9.3 billion gallons of water and 162 million kWh of energy, respectively. In contrast, the negative externalities caused by manufacturers' traditional production should be corrected by environmental taxes, such as the NOx tax in

Sweden, the landfill tax in the UK and the new vehicle tax in Catalonia. To show the applicability of the proposed model, we design a numerical example using the data from Johari et al. (2019), which are $\alpha = 50$, $\beta = 1$, $\gamma = 0.2$, $c = 5$, $k_1 = 2$, $k_2 = 1$, $k_3 = 1$, $\theta = 0.001$, $\Delta\theta = 0.001$, $\Delta c = 1$, $\mu_0 = 0.1$.



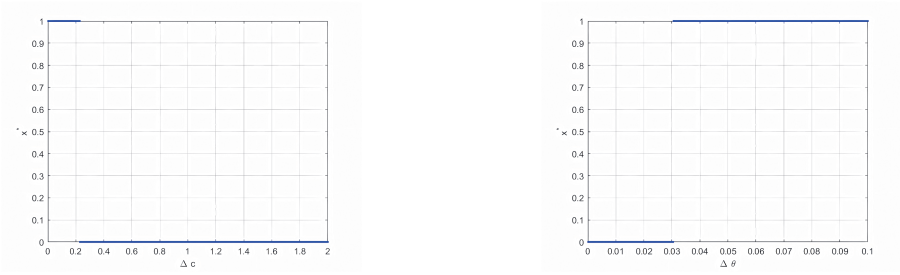
(a) (b)

Figure 2.2: The state variable x changes according to time t.

Fig. 2.2 (a) shows that the equilibrium will converge to $x^* = 0$ when the initial value of x is in $[0, 1)$ at $t = 0$, and converge to $x^* = 1$ when the initial value of x is 1. Fig. 2.2 (b) displays the line-element field of the CSR level, which reflects the evolution path of state variable $x(t)$ to the x^* with time t. Fig. 2.2 shows that regardless of the initial state in the market, the manufacturer population must eventually reach a stable state in dynamic competition. On the one hand, it shows that a market must move towards stable operation in the long run even though the manufacturers are bounded rational. On the other hand, it also implies that due to self-interest, manufacturers must evade social responsibility when the government's punishment is lighter.

As shown in Fig. 2.3 (a), the CSR level is maintained at 1 when Δc is less than 0.23. When the extra cost exceeds 0.23, manufacturers would lack motivation to produce CSR goods. It indicates that the government can encourage manufacturers to reduce additional costs and elevate the CSR level by developing green technologies.

It can be observed in Fig. 2.3 (b) that the CSR level changes from 0 to 1



(a) (b)

Figure 2.3: The CSR level x^* changes with respect to Δc and $\Delta\theta$.

as $\Delta\theta$ increases to 0.03. It signifies that the manufacturer's selection of the strategy T becomes unprofitable once the taxes surpass a specific threshold, aligning the manufacturer's financial objective with the environmental objective. That is to say, lighter tax penalties do not contribute to increasing the CSR level, whereas only high environmental taxes can incentivize firms to embrace the CSR strategy. However, high environmental taxes can potentially result in negative implications of social welfare, the economy, and politics. For example, Canada's environmental tax policy has led to rising prices and a sharp climb in unemployment. Residents in Ottawa will be paying more than $1,000 extra each year to comply with the carbon tax laws (Institute, 2016). Moreover, the environmental tax can be politically difficult. Initiative 1631 proposed a carbon tax of $15 per metric ton of carbon dioxide emitted. In the Washington State Referenda, Initiative 1631 was voted down, 56 percent to 44 percent (McGillis, 2019). In a similar case in Australia, the Labor Party came to power and launched a referendum on an environmental tax, ending a two-year carbon-pricing scheme in July 2014 (McGillis, 2019).

Therefore, other policies are needed beyond environmental taxes. We proceed to discuss the influence of media supervision on the evolutionary behaviors of the manufacturer population.

2.4 Media supervision mechanism

This section considers a scenario with media supervision, where the media influences consumers' behaviors by regulating manufacturers and exposing their strategies. Therefore, the demand and payoff of manufacturers are related to the supervising strategy of media, i.e. the effort level of supervision and exposure($\rho \in [0,1]$). Significantly, The profit of media is regulated by the evolutionarily stable strategy of the manufacturer population rather than by the actions of individual manufacturers. In other words, the media only benefits from the exposure of the strategy C or strategy T, regardless of which firm it is.

The specific game sequence is explained below. **Step 1:** The media determines an optimal effort level to maximize profit. **Step 2:** Under the optimal effort level, the manufacturer population learns and updates strategies until reaching the evolutionary stable state.

According to the game structure, the game of the manufacturer population under media supervision can be described as a Stackelberg game embedded with an evolutionary game, in which the media is the leader and the manufacturer population are followers. The definition of the Stackelberg evolutionary game is,

Definition 2.6. *A Stackelberg evolutionary game model of the manufacturer population under media supervision is defined as* $G^m =< N, m, S, \rho, X, \Pi^m, U >$*, where*

(a) m *represents the media (leader), and* $N = [0,1]$ *is the population of manufacturers (followers);*

(b) $S = \{C, G\}$ *is the set of firms' strategies;*

(c) $\rho \in [0,1]$ *is the effort level of the media;*

(d) $X = \{(x_C(\rho), x_T(\rho)) \in \mathbb{R}^2_+ : x_C(\rho) + x_T(\rho) = 1, 0 \leq x_C(\rho) \leq 1\}$ *is the set of strategy distributions under the media's strategies;*

(e) $\Pi^m : S \times X \to \mathbb{R}^2$ *is the expected payoff function;*

(f) $U : \rho \times X \to \mathbb{R}$ *is a function mapping each* $\rho \in [0,1]$ *and* $x \in [0,1]$ *to the profit of media.*

We first analyze the strategy and profit function of the leader.

2.4.1 Media's strategy and profit

As mentioned above, the level of effort on supervision and exposure $\rho \in [0,1]$ is the strategy of media. The higher the effort level ρ, the greater the chance of exposure when the manufacturer chooses T strategy. Assuming that the supervision cost is a quadratic function of ρ with cost coefficient k_3 ($k_3 > 0$).

Real cases show that the media gains great social attention by exposing corporate behaviors. For example, the dieselgate incident has made the automotive energy field continue to receive attention since its exposure, bringing great traffic to media reports (Baumgärtner and Letmathe, 2020). Mass media used the environmental documentary "Under the Dome" to introduce the haze pollution of Chinese firms through the mass media and got more than 200 million views within two days after the film was released (Tu et al., 2020). Then the media can turn this Web traffic into cash through advertising placement, public relations and platform sharing. Other evidence suggests that media coverage of CSR compliance behaviors also generates social attention, which is generally lower than coverage of pollution. Thus, we use k_2 to represent the coefficient of media's profit when manufacturers follow strategy C. Let k_1 ($k_1 > k_2$) measure the unit media's profit when manufacturers use strategy T and are successfully exposed. We consider the media's profit in the evolutionarily stable state, the profit function is constructed as below:

$$U = k_1\rho(1-x) + k_2x - k_3\rho^2. \tag{2.10}$$

As a for-profit supervisory organization, the goal of the media is to maximize profits by optimizing decisions, which will affect the evolution of manufacturers. Conversely, Eq. (2.10) shows that media profit is also influenced by the ESS of the firm population. In what follows, we analyze the demand and evolution of the manufacturer population under media supervision.

2.4.2 Evolutionary game of the manufacturer population with media supervision

The media can influence public judgment and thus consumers' willingness to pay. Positive media coverage boosts a company's social image, while news about pollution weakens consumers' purchasing power (Nicolas et al., 2024). Next, we discuss the form of demand function under media supervision.

Due to the comparison of different strategies, media supervision will lead to the scaling-down of demand for strategy T and scaling-up of demand for strategy C in the case of TC/CT. In the strategy profile TT, the demand will reduce proportionally because consumers are informed that the vast majority of products are produced through traditional strategies, causing green consumers to lose confidence in the product due to spillover effect (Awasthy et al., 2022). The demand function under strategy structure CC is the same as in Section 2.3 because no strategy is disclosed as a traditional strategy.

Compared with Eq. (2.4), let $d_{S_iS_j}(\rho)$, $S_i, S_j \in \{C, T\}$ denote the demand of a manufacturer with strategy S_i against strategy S_j under media supervision, the demand function becomes heterogeneous in each strategy profile:

$$d_{S_iS_j}(\rho) = \begin{cases} q_{CC} & S_iS_j = CC, \\ q_{CT} + \rho\mu & S_iS_j = CT, \\ q_{TC} - \rho\mu & S_iS_j = TC, \\ q_{TT}(1 - \rho\mu) & S_iS_j = TT, \end{cases} \tag{2.11}$$

where μ is the green sensitivity of consumers after media exposure.

Under the demand function Eq. (2.11), the objective function of the strategy profile CC changes to

$$\begin{cases} \max\limits_{p_C} \Pi_{CC} = (p_C - c - \Delta c) d_{CC}(\rho), \\ \max\limits_{p_C} \Pi_{CC} = (p_C - c - \Delta c) d_{CC}(\rho). \end{cases} \tag{2.12}$$

The goal of each firm is shown as follows when one firm uses strategy C and another firm uses strategy T.

$$\begin{cases} \max\limits_{p_C} \Pi_{CT} = (p_C - c - \Delta c) d_{CT}(\rho), \\ \max\limits_{p_T} \Pi_{TC} = (p_T - c) d_{TC}(\rho). \end{cases} \tag{2.13}$$

In the strategy profile TT, the optimization problems faced by the two firms turn into

$$\begin{cases} \max\limits_{p_T} \Pi_{TT} = (p_T - c) d_{TT}(\rho), \\ \max\limits_{p_T} \Pi_{TT} = (p_T - c) d_{TT}(\rho). \end{cases} \tag{2.14}$$

By calculating the above optimization problems, we can get the payoff of each strategy profile under media supervision, which is recorded as the following payoff table,

Table 2.2: The payoff table with media supervision

	Strategy T	Strategy C
Strategy T	Π^m_{TT}	Π^m_{TC}
Strategy C	Π^m_{CT}	Π^m_{CC}

The value of $\Pi^m_{S_i S_j}$, $S_i, S_j \in \{C, T\}$ and corresponding optimal price (demand) are detailed in Subsection 2.7. Then we give the evolution-

arily stable strategy (ESS) with media supervision according to Table 2.2.

$$\begin{cases} x^*(\rho) = 0 & \text{if } \Pi^m_{CC} < \Pi^m_{TC},\ \Pi^m_{TT} > \Pi^m_{CT}, \\ x^*(\rho) = 1 & \text{if } \Pi^m_{CC} > \Pi^m_{TC},\ \Pi^m_{TT} < \Pi^m_{CT}, \\ x^*(\rho) = 0, 1 & \text{if } \Pi^m_{CC} > \Pi^m_{TC},\ \Pi^m_{TT} > \Pi^m_{CT}, \\ x^*(\rho) = \frac{\Pi^m_{TT} - \Pi^m_{CT}}{\Pi^m_{CC} + \Pi^m_{TT} - \Pi^m_{CT} - \Pi^m_{TC}} & \text{if } \Pi^m_{CC} < \Pi^m_{TC},\ \Pi^m_{TT} < \Pi^m_{CT}. \end{cases} \tag{2.15}$$

Eq. (2.15) show the ESS with payoff matrix in Table 2.2 under different conditions.

After clarifying the form of each player's strategy and profit, we analyze the equilibrium of the Stackelberg evolutionary game theoretically in the next section.

2.4.3 Analysis of the Stackelberg evolutionary equilibrium

To introduce the definition of the Stackelberg evolutionary equilibrium, we first propose the concept of optimal response for the manufacturer population (followers) under each media's (leader's) strategy $\rho \in [0, 1]$.

Definition 2.7. *In a Stackelberg evolutionary game G^m with two strategies of the manufacturer population, the set $X^*(\rho) \in [0, 1]^2$ defined for each $\rho \in [0, 1]$ by*

$$X^*(\rho) = \{\boldsymbol{x}^*(\rho) = (x^*(\rho), 1 - x^*(\rho)) : x^*(\rho) \text{ is the ESS } \}$$

is the optimal response set of the manufacturer population to the strategy $\rho \in [0, 1]$ of media.

According to Definition 2.7, the optimal response of the manufacturer population represents the CSR level reached by the dynamic evolution of manufacturers after the media determines the strategy ρ.

Definition 2.8. *A strategy $\rho^* \in [0,1]$ is called a Stackelberg evolutionary equilibrium for the media if*

$$\min_{\boldsymbol{x}^* \in X^*(\rho^*)} U(\rho^*, \boldsymbol{x}^*) = \max_{\rho \in [0,1]} \min_{\boldsymbol{x}^* \in X^*(\rho)} U(\rho, \boldsymbol{x}^*).$$

Combined with the optimal response of followers in Definition 2.7 and the ESS with media supervision (2.15), the objective function of the media is

$$\max_{\rho \in [0,1]} U = k_1 \rho (1 - x^*(\rho)) + k_2 x^*(\rho) - k_3 \rho^2. \tag{2.16}$$

Under the definition of the Stackelberg evolutionary equilibrium, we analyze the existence and uniqueness of the equilibrium by the following two theorems.

Proposition 2.9. *For the ESS (2.15), the CSR level $x^*(\rho)$ is nondecreasing with respect to the effort level ρ if*

$$\alpha > max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\} \tag{2.17}$$

where

$$\begin{cases} \alpha_1 = (c + \Delta c)(\beta - \gamma) - \mu_0, \\ \alpha_2 = c(\beta - \gamma) + \mu_0, \\ \alpha_3 = \frac{(2\beta^2 - \gamma^2)(c + \Delta c) - (2\beta - \gamma)\mu_0 - \beta\gamma c}{(2\beta + \gamma)}, \\ \alpha_4 = \frac{(2\beta^2 - \gamma^2)c + (2\beta - \gamma)\mu_0 - \beta\gamma(c + \Delta c)}{(2\beta + \gamma)}. \end{cases}$$

A supplementary remark is given on the nondecreasing of $x^*(\rho)$ in Proposition 2.9. As we can see from the proof, $x^*(\rho)$ monotonically

increases when $x^* \in (0,1)$ and remains constant only at $x^* = 0/1$. The parameter condition is general in terms of economic significance, as it ensures that the demand functions are greater than 0.

Proposition 2.9 reveals that CSR level x^* is nondecreasing with exposure degree ρ through reducing the relative payoffs of T strategy, that is, the optimal response of the manufacturer population is nondecreasing with the strategy of the media. This means that the CSR level can be maintained at a high value through media supervision.

In the information age, the media plays an increasingly important role in information dissemination and enterprise supervision. Especially in sustainable development, media coverage can change the public's perception of the environment and pollution, prompting companies to take more proactive action on environmental protection. For instance, a large amount of empirical data of Chinese firms shows that high-media exposure makes the firms' commitment of CSR significantly higher at 1% than low-media exposure (Tavakolifar et al., 2021). Another practical case is that after the Volkswagen scandal, increasing media attention has accelerated the shift from diesel to new energy vehicles, which achieved a significant increase in the CSR level across the automotive industry (Awasthy et al., 2022).

In the following theorem, we analyze the existence and uniqueness of the media's optimal strategy under the ESS.

Theorem 2.10. *For the profit function* (2.10) *and every ESS* (2.15) *satisfying Eq.* (2.17)*, there exists a unique Stackelberg evolutionary equilibrium* $\rho^* \in (0,1]$ *for the media.*

Theorem 2.10 indicates media has a unique optimal strategy ρ^* corresponding to a maximum profit U, which reveals the motives of media regulation in practice. As the example in Subsection 2.4.1 shows, more regulatory effort leads to more traffic, which means the increase in the media's profit. On the other hand, the marginal cost of media exposure is constantly rising. This is due to the difficulty of investigating firms,

and to add insult to injury, some agencies may help firms conceal non-CSR activities, such as the Sanlu milk powder scandal in China. Thus, if the regulatory level ρ is greater than the optimal value ρ^*, the media will benefit from large traffic but will also have to spend more. To maximize the profit, the media should select the optimal strategy ρ^* according to the market characteristics.

In combination with Proposition 2.9, media interests are generally consistent with the improvement of the CSR level. The optimal strategy of the media can achieve a high CSR level at the same time, thus achieving a win-win situation of profitability and environmental performance.

Corollary 2.11. *Supposing that $\rho^* \in [0, 1]$ is the Stackelberg evolutionary equilibrium for the media, the manufacturer population has a unique optimal response x_m^* for ρ^*.*

Corollary 2.11 shows that there is also a unique optimal CSR level (denoted by x_m^*) under the media's optimal strategy, which ensures the effectiveness of media in market regulation. The uniqueness of game equilibrium with media supervision reduces the possibility of industry chaos to a certain extent, which is conducive to the smooth and orderly market operation.

For an implicit function $f(\rho^*, \alpha, \beta, \gamma, c, \theta, \Delta\theta, \Delta c, \mu_0, \mu, k_1, k_2, k_3, k_4) = 0$, ρ^* can be seen as a solution to this equality. Theorem 2.10 and Corollary 2.11 indicate the existence and uniqueness of ρ^* and x_m^*, but the parameter expression is relatively complex. Therefore, the optimal value is calculated by numerical simulation. Now we give an algorithm to solve the equilibrium for the Stackelberg evolutionary game first.

According to the algorithm below, we calculate the optimal value under the given parameters by a numerical example in the next subsection.

Algorithm 1 Framework of numerical solution for the Stackelberg evolutionary equilibrium

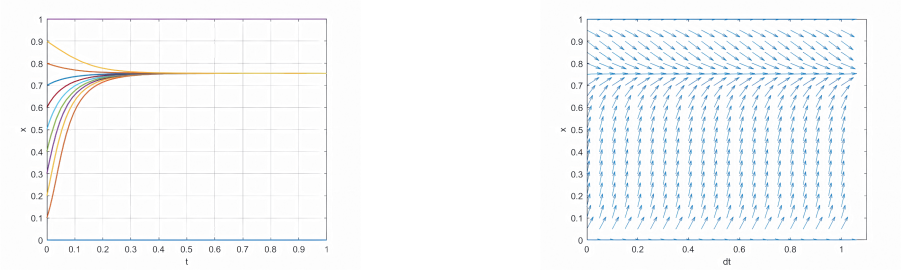
Input: parameters $\alpha, \beta, \gamma, c, \theta, \Delta\theta, \Delta c, \mu_0, \mu, k_1, k_2, k_3, N$
Output: the Stackelberg evolutionary equilibrium ρ^*, x_m^*, media's maximum profit U^*

$U^* \leftarrow 0$;
for $i = 1 \rightarrow N$ **do**
  $\rho = \frac{i-1}{N-1}$;
  calculate the payoff of each strategy profile $\Pi_{S_i S_j}^m$, $S_i, S_j \in \{C, T\}$;
  according to Eq. (2.15), determine and calculate the ESS x^*;
  calculate the media's profit U according to Eq. (2.10);
  if $U > U^*$ **then**
    $U^* \leftarrow U, x_m^* \leftarrow x^*, \rho^* \leftarrow \rho$;
  end if
end for

According to the above algorithm, we calculate the optimal value under the given parameters by a numerical example in the next subsection.

2.4.4 Numerical simulations

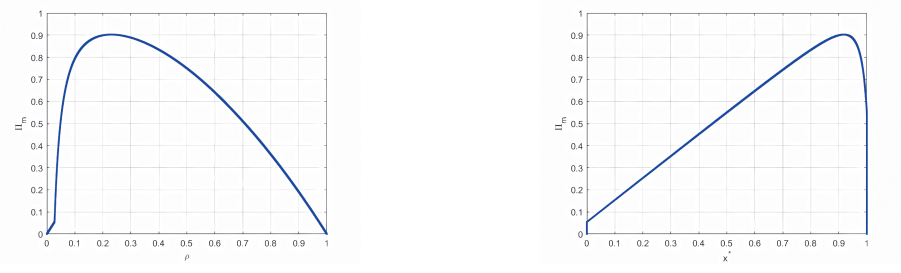
Continuing the example in Subsection 2.3.3, we consider media exposure to manufacturers. A case in point is the concept car introduced by Volkswagen that runs on the latest hydrogen fuel cell technology, which produced nitrogen oxide emissions about 40 times over the limit. After the incident became public, Volkswagen suffered a massive crisis of trust and reputation, with its share price plunging 30% and its market value losing 35 billion euros. In this subsection, we fix the parameters $\alpha = 50$, $\beta = 1$, $\gamma = 0.2$, $c = 5$, $\theta = 0.001$, $\Delta\theta = 0.001$, $\Delta c = 1$, $\mu_0 = 0.1$, $\mu = 1$, $k_1 = 2$, $k_2 = 1$, $k_3 = 1$.



(a) (b)

Figure 2.4: The state variable x changes according to time t.

Fig. 2.4 (a) shows that the equilibrium will converge to $x^* = 0.74$, wherever the initial value is in $(0, 1)$ under the parameter $\rho = 0.1$. In other words, the proportion of firms using strategy C will be stable at 74% in the long run. Despite a tenfold reduction in the environmental tax compared to Section 2.3.3, supervision and exposure change the CSR level from 0 to 0.74. Fig. 2.4 (b) displays the line-element field of CSR level, which reflects the evolution path of state variable $x(t)$ to the ESS (x^*) with time t under different initial values.

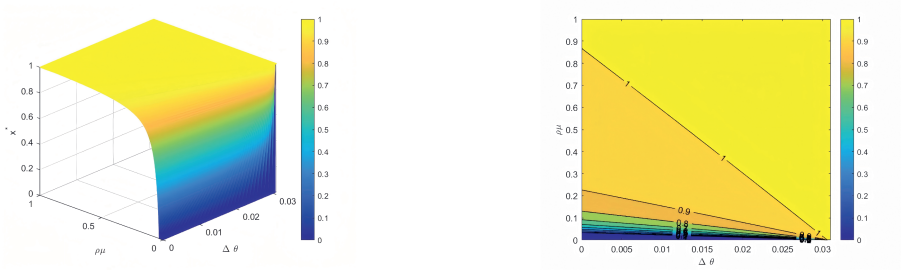


(a) (b)

Figure 2.5: The media's profit change with respect to ρ and x^*.

The monotonicity of x^* and the existence and uniqueness of ρ^* have already been shown in the previous subsection. Since the analytical solution of ρ^* is difficult to express, Fig. 2.5 shows the maximum value

of media profit and the corresponding optimal effort level ρ^* and optimal CSR level x^* through the numerical method. As shown in Fig. 2.5 (a), the changes of media profit with respect to effort level ρ are convex, and the optimal point is $(\rho^*, U^*) = (0.23, 0.90)$. Fig. 2.5 (b) supports the analysis in Subsection 2.4.2, where U monotonically increases when $x^* < 0.91$ and decreases when $x^* > 0.91$. We can conclude that the optimal point is $(\rho^*, x_m^*, U^*) = (0.23, 0.91, 0.90)$.



(a) (b)

Figure 2.6: Substitution between ρ and $\Delta\theta$ about the CSR level x^*.

Fig. 2.6 (a) shows a three-dimensional diagram reflecting the combined effects of media supervision $\rho\mu$ and environmental tax $\Delta\theta$ on the CSR level x^*. According to Fig. 2.6 (b), the coordinate plane XOY represents the substitution of $\rho\mu$ and $\Delta\theta$ at the same CSR level. In addition, any combination of $\rho\mu$ and $\Delta\theta$ can achieve a relatively high level of CSR in the range of approximately $(0.2, 1) \times (0.03, 1)$. Fig. 2.6 indicates that the government can improve the CSR level through media supervision instead of environmental taxes.

It is well known that unreasonable or high environmental taxes would increase deadweight losses and hinder economic development. Take a practical case, Columbia's analysis showed that by 2030 the environmental tax would have increased average electricity prices by as much as 10 percent and gasoline prices by nearly 10 cents per gallon, bringing down GDP by about two-tenths of a percent (Institute, 2016). Another example is that many businesses may transfer jobs and resources

away from the local economy, which shifted about $21 trillion in total assets according to a British survey (McGillis, 2019). As an alternative policy tool, the media can not only effectively regulate the market by reducing information asymmetry, but also transform the economy from rapid growth to high-quality development.

2.5 Discussion of extensions

2.5.1 Evolutionary game model with dynamic green sensitivity

We consider extending the green sensitivity to the dynamic case and present a numerical method to learn the optimal strategy for media in the evolutionarily stable state.

To determine the change of green sensitivity over time, we first analyze the impact of consumers' green preference for products. According to policies and public opinion on sustainability, consumer greenness is increasing over time. What's more, the more green manufacturers there are, the more sensitive consumers will be to green goods. And the growth rate of μ will decrease as μ and x increase. Assuming a continuous time system ($t \in (0, \infty)$), the differential equation describing the gradient learning of effort level over time is as follows,

$$\dot{\mu} = \mu(1 - \mu - v_1 x + v_2 \rho). \tag{2.18}$$

where v_1 and v_2 measure the the restraining effect and the promoting effect of $x(t)$ and $\mu(t)$ on the growth rate of μ.

To be specific, they can use the local estimation of marginal profit to infer the effort level at the next period, in which the effort level rises as marginal profit increases and declines conversely. Combined with the replicator dynamic equation of the CSR level x, a nonlinear system of differential equations of the dynamic green sensitivity and the CSR

level is

$$\begin{cases} \dot{\mu} = \mu(1-\mu-v_1 x+v_2\rho), \\ \dot{x} = x(1-x)[(\Pi_{CC}^m+\Pi_{TT}^m-\Pi_{CT}^m-\Pi_{TC}^m)x+\Pi_{CT}^m-\Pi_{TT}^m]. \end{cases} \tag{2.19}$$

The dynamic green sensitivity μ and the CSR level x influence each other and both of them are regulated by media strategy. Does the media have the optimal strategy and the corresponding maximum profit under the dynamic system (2.19)? Then we analyze this problem by numerical method and give a algorithm first.

Algorithm 2 Framework of numerical solution for the nonlinear system

Input: parameters $\alpha, \beta, \gamma, c, \theta, \Delta\theta, \Delta c, \mu_0, k_1, k_2, k_3, N$
Output: the optimal point $(\rho^*, \mu_m^*, x_m^*, U^*)$
1: $U^* \leftarrow 0$;
2: **for** $i = 1 \rightarrow N$ **do**
3: $\rho = \frac{i-1}{N-1}$;
4: calculate the equilibrium point (μ^*, x^*) of the system (2.19) based on Runge-Kutta Method;
5: calculate the media's profit U according to Eq. (2.10);
6: **if** $U > U^*$ **then**
7: $U^* \leftarrow U, x_m^* \leftarrow x^*, \mu_m^* \leftarrow \mu^*, \rho^* \leftarrow \rho$;
8: **end if**
9: **end for**

For simplicity, we assume a zero tax in this subsection, which will not affect the general outcome of the game between the media and manufacturer population. In the example from Section 2.3, we fix the parameters $\alpha = 50$, $\beta = 1$, $\gamma = 0.2$, $c = 5$, $\theta = 0.001$,$\Delta\theta = 0.001$, $\Delta c = 1$, $\mu_0 = 0.1$, $v_1 = 1$, $v_2 = 1$, $k_1 = 2$, $k_2 = 1$, $k_3 = 1$.

In the numerical example, Fig. 2.7 (a) and (b) show the variable $x(t)$ and $\mu(t)$ change over time under parameter ρ. With the increase of ρ, the CSR level $x(t)$ increases, and the green sensitivity $\mu(t)$ decreases

first and then increases. From Fig. 2.7 (c), we can observe that the media profit U and the effort level ρ are inverted U-shaped relationships when green sensitivity is dynamic. The optimal effort level ρ^* is about $\rho^* = 0.31$ and the corresponding value is $(x_m^*, \mu_m^*, U^*) = (0.83, 0.47, 0.84)$. Finally, Fig. 2.7 (d) shows the evolution of the differential equation system (2.19) over time under the optimal effort level.

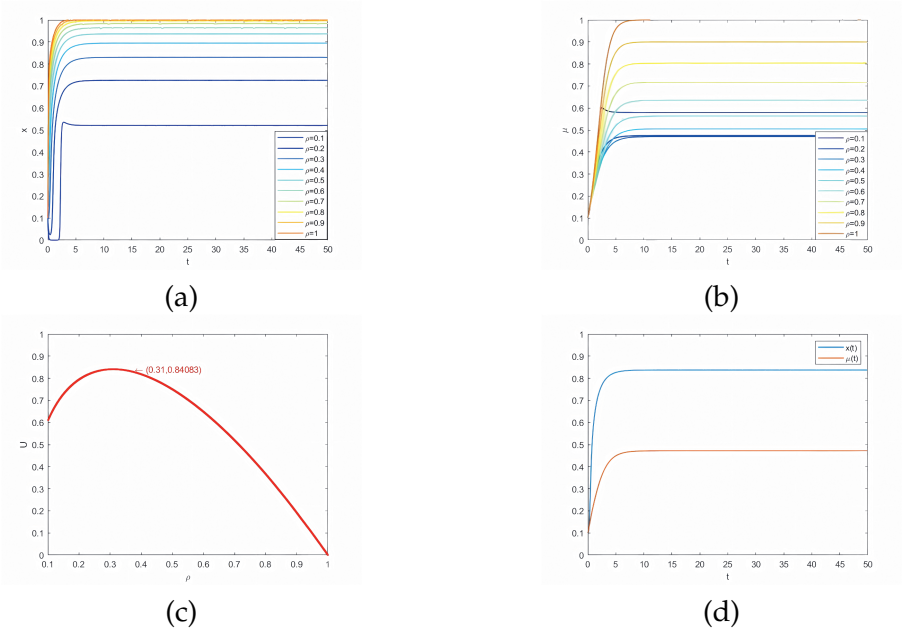


Figure 2.7: The differential equation system (2.19).

2.5.2 Manufacturers competition with heterogeneous costs

In this subsection, the model is extended to heterogeneous cases. Suppose there are two types of manufacturers with different costs in the market, one with high production cost (c_h) and the other with low production cost (c_l). Similar to the previous sections, each firm can

choose the traditional strategy (with costs c_h, c_l) or the CSR strategy (with costs $c_h + \Delta c, c_l + \Delta c$).

Since the demand is not affected by the cost, the expression of the demand function is still Eq. (2.11). In the heterogeneous case, we get the payoff matrix in Tables 2.3-2.6, whose expressions are attached in 2.7.

Table 2.3: h & l

	T	C
T	Π_{TT}^{hl}	Π_{TC}^{hl}
C	Π_{CT}^{hl}	Π_{CC}^{hl}

Table 2.4: l & h

	T	C
T	Π_{TT}^{lh}	Π_{TC}^{lh}
C	Π_{CT}^{lh}	Π_{CC}^{lh}

Table 2.5: h & h

	T	C
T	Π_{TT}^{hh}	Π_{TC}^{hh}
C	Π_{CT}^{hh}	Π_{CC}^{hh}

Table 2.6: l & l

	T	C
T	Π_{TT}^{ll}	Π_{TC}^{ll}
C	Π_{CT}^{ll}	Π_{CC}^{ll}

Next, we construct an evolutionary game model of heterogeneous manufacturers under media supervision. We assume that the proportion of high-cost manufacturers and low-cost manufacturers in the market is p and $1-p$. Assuming that x is the percentage of strategy C in the manufacturer population with high cost, the expected payoffs of high-cost manufacturers are expressed as:

$$\begin{aligned}
\Pi_C^h &= p(1-p)[xy\Pi_{CC}^{hl} + x(1-y)\Pi_{CT}^{hl}] + p^2[x^2\Pi_{CC}^{hh} + x(1-x)\Pi_{CT}^{hh}], \\
\Pi_T^h &= p(1-p)[(1-x)y\Pi_{TC}^{hl} + (1-x)(1-y)\Pi_{TT}^{hl}] \\
&\quad + p^2[x(1-x)\Pi_{TC}^{hh} + (1-x)(1-x)\Pi_{TT}^{hh}].
\end{aligned}$$

Assuming that y is the percentage of strategy C in the manufacturer population with low cost, the expected payoffs of low-cost manufacturers are expressed as:

$$\begin{aligned}
\Pi_C^l &= p(1-p)[xy\Pi_{CC}^{lh} + y(1-x)\Pi_{CT}^{lh}] + (1-p)^2[y^2\Pi_{CC}^{ll} + y(1-y)\Pi_{CT}^{ll}], \\
\Pi_T^l &= p(1-p)[(1-y)x\Pi_{TC}^{lh} + (1-x)(1-y)\Pi_{TT}^{lh}] \\
&\quad + (1-p)^2[y(1-y)\Pi_{TC}^{ll} + (1-y)(1-y)\Pi_{TT}^{ll}].
\end{aligned}$$

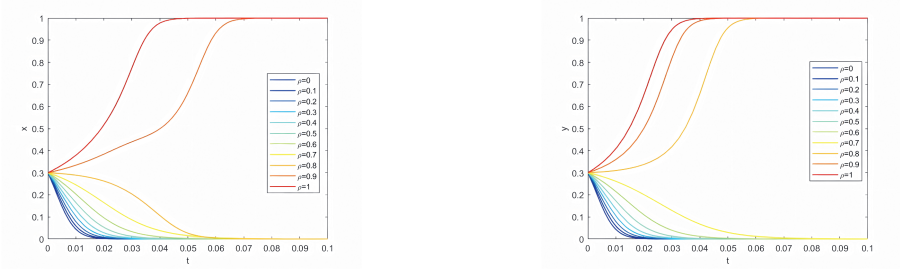
The replicator dynamics equation describing the evolution of percentages of each strategy over time is changed to:

$$\begin{cases} \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)(1-x(t))(\Pi_C^h - \Pi_T^h), \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)(1-y(t))(\Pi_C^l - \Pi_T^l). \end{cases}$$

In the heterogeneous case, the payoffs gained by the media through the supervision of the two types of firms are as follows:

$$\begin{cases} U_x = k_1\rho(1-x) + k_2 x - k_3\rho^2, \\ U_y = k_1\rho(1-y) + k_2 y - k_3\rho^2. \end{cases}$$

In what follows, we analyze the characteristics of the market under the heterogeneous case through numerical experiments. We set the same parameters as in the previous sections, which are $\alpha = 50$, $\beta = 1$, $\gamma = 0.2$, $\theta = 0.001$,$\Delta\theta = 0.001$, $\Delta c = 1$, $\mu_0 = 0.1$, $k_1 = 2$, $k_2 = 1$, $k_3 = 1$. The heterogeneous costs are fixed as $c_h = 6$ and $c_l = 4$.

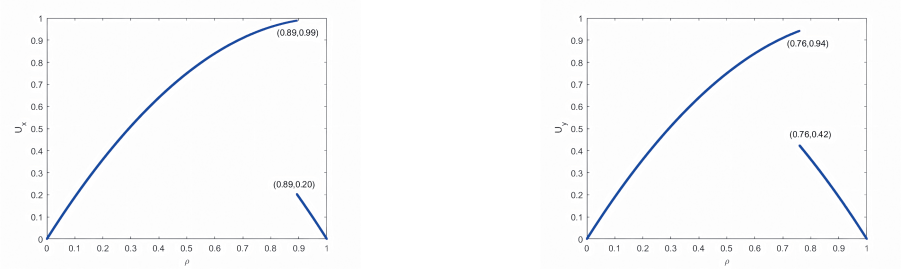


(a) (b)

Figure 2.8: Trajectories for the CSR level over time under different values of ρ.

From Fig. 2.8, the CSR level is generally stable at the boundary point in the heterogeneous case, and it changed from 0 to 1 when ρ reached a certain value. Comparing Fig. 2.8 (a) and (b), it can be observed that when ρ arrives at 0.8, the CSR level of low-cost manufacturers reaches

1, while the CSR level of high-cost manufacturers remains at 0. This indicates that the threshold for effective supervision falls as the cost decreases.

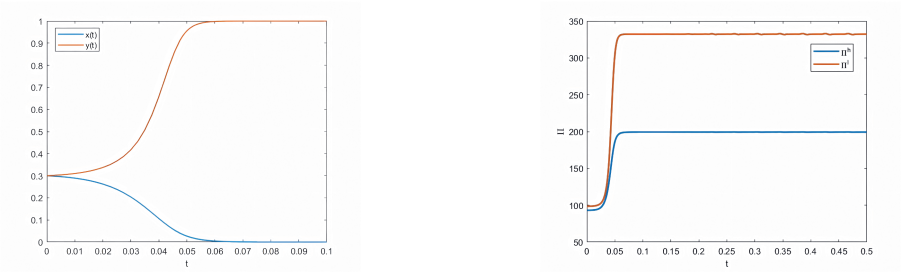


(a) (b)

Figure 2.9: The media's profit change with respect to ρ.

Fig. 2.9 shows that the media's payoff still satisfies the property of Theorem 2.10, that is, the media has an optimal regulatory strategy when supervising each type of manufacturer. However, in heterogeneous situations, there may be a discrepancy between the goal of maximizing media benefits and achieving environmental objectives. As shown in the figure, the CSR level is 0 when the media gets the maximum payoff. Therefore, the effective regulation, which makes the CSR level reach 1, will result in a payoff loss for the media. That is to say, some regulatory subsidies should be paid to the media to prevent a decline in its payoff and the failure of regulation.

By fixing $\rho = 0.8$, the evolution of the CSR level x and y over time is simulated in Fig. 2.10 (a). And Fig. 2.10 (b) is the evolution of the payoff for each type of manufacturer over time. It is evident that low-cost manufacturers not only have a high CSR level, but also can get a greater payoff. Thus, to getting more profits, manufacturers are driven to engage in technical innovation to reduce production costs. For environmental purposes, the government also has an incentive to assist manufacturers in lowering their manufacturing costs. Therefore, many manufacturers ultimately produce goods at essentially the same cost,



(a) (b)

Figure 2.10: The evolution of the CSR level and profit for the two types of manufacturers.

which refers to an industry with homogeneous competition. For example, Tesla's electric vehicles use clean energy instead of traditional fuels, which has led to a wave of innovation in the new energy vehicle industry (Zheng et al., 2022). To obtain more profits, auto manufacturers imitate each other in technology and production, which makes new energy products become homogenized while their cost is reduced.

2.6 Conclusion

Despite the advantages of environmental taxation, taxes alone may create distortions within the economy that prevent the occurrence of optimal actions. For example, tax restrictions on information dissemination tend not to alter consumer behavior and patterns for a period of time. In such circumstances, additional policy tools may be needed to provide an optimal instrument mix. Based on the influence of media on consumer demand, this paper proposes a media supervision mechanism through the Stackelberg evolutionary game. In this mechanism, we discuss the cost structure and revenue mode of the media, as well as the heterogeneous influence of media on the demand of manufacturers' strategy profiles. To maximize the benefits of both the media and the manufacturer population, we analyze the equilibrium of this

game and draw some conclusions.

Our results show that the CSR level increases with the effort level of media supervision. In other words, under media supervision, the media supervision mechanism aligns the sustainable goals and the firm's goals by guaranteeing socially concerned firms higher profit compared to traditional strategies, thus encouraging manufacturers to use the CSR strategy. Green development requires manufacturers to consider both internal control and external supervision to drive technological innovation while meeting social review standards.

Another research result demonstrates that the media has a unique optimal strategy in the Stackelberg evolutionary equilibrium from commercial considerations. This research implies that it would be beneficial for the government to provide subsidies and regulations for the media to increase the exposure degree, as well as to strengthen environmental education and publicity to enhance green consumption awareness.

As shown in Fig. 2.4 and Fig. 2.6, media supervision serves as a substitute for environmental taxes in enhancing the CSR level. The green technology reform of manufacturers can be driven by the organic connection of environmental taxes, media exposure, and consumer awareness. Governments can employ multi-party regulation to replace high taxes, thereby reducing deadweight losses and maintaining market stability.

In Section 2.5, we have also discussed how firms' payoff and equilibrium vary with respect to exogenous parameters. It is found that the CSR level is positively correlated with environmental taxes, green sensitivity, and media unit profit, and negatively correlated with the additional costs of strategy C and supervision costs. This implies that it is important to understand how businesses operate, consumer markets, and the way stakeholders benefit before formulating policies.

We acknowledge some limitations in our work and suggest several directions for future research. First, media bias is not taken into account,

and it is interesting to study the collusive behavior of the media and some manufacturers. Secondly, we do not quantify the government's payoff function, thereby not getting the specific expression of the optimal tax rate. When the government designs environmental tax, the optimal tax rate of society deserves further study. Finally, we suggest that future research could be extended to the dynamic media supervision mechanism, such as the update strategy of the media.

2.7 Appendix

Proof of Proposition 2.5

Proof. We only have to analyze the case where $\Pi^w_{CC} - \Pi^w_{TC} < 0$ and $\Pi^w_{TT} - \Pi^w_{CT} < 0$. Take the partial derivatives of parameter $\Delta\theta$ of x^*, $\frac{\partial x^*}{\partial \Delta\theta}$ equals to

$$\frac{(\Pi_{TT} - \Pi_{TC})(\Pi^w_{TT} - \Pi^w_{CT}) - \Pi_{TT}(\Pi^w_{CC} + \Pi^w_{TT} - \Pi^w_{CT} - \Pi^w_{TC})}{(\Pi^w_{CC} + \Pi^w_{TT} - \Pi^w_{CT} - \Pi^w_{TC})^2}.$$

Eq. (2.20) can be easily calculated as

$$\frac{\partial x^*}{\partial \Delta\theta} = \frac{-\Pi_{TT}(\Pi^w_{CC} - \Pi^w_{TC}) - \Pi_{TC}(\Pi^w_{TT} - \Pi^w_{CT})}{(\Pi^w_{CC} + \Pi^w_{TT} - \Pi^w_{CT} - \Pi^w_{TC})^2}. \tag{2.20}$$

From the conditions $\Pi^w_{CC} - \Pi^w_{TC} < 0$ and $\Pi^w_{TT} - \Pi^w_{CT} < 0$, it holds that $\frac{\partial x^*}{\partial \Delta\theta} > 0$, which means that CSR level x^* increases as the increment of $\Delta\theta$.

Proof of Proposition 2.9

Proof. From Table 2.9, $\Pi^m_{TC} - \Pi^m_{CC}$ and $\Pi^m_{TT} - \Pi^m_{CT}$ decrease with respect to ρ. According to $Eq.(2.15)$, we just have to show the CSR level x^* increases with effort level ρ when $\Pi^m_{CC} - \Pi^m_{TC} < 0$ and $\Pi^m_{TT} - \Pi^m_{CT} < 0$.

The first derivative of x^* with respect to ρ yields that

$$\begin{aligned}\frac{\partial x^*}{\partial \rho} &= \frac{\frac{\partial(\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho}(\Pi^m_{CC}-\Pi^m_{TC}+\Pi^m_{TT}-\Pi^m_{CT})}{(\Pi^m_{CC}-\Pi^m_{TC}+\Pi^m_{TT}-\Pi^m_{CT})^2} \\ &- \frac{\frac{\partial(\Pi^m_{CC}-\Pi^m_{TC}+\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho}(\Pi^m_{TT}-\Pi^m_{CT})}{(\Pi^m_{CC}-\Pi^m_{TC}+\Pi^m_{TT}-\Pi^m_{CT})^2} \\ &= \frac{\frac{\partial(\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho}(\Pi^m_{CC}-\Pi^m_{TC})-\frac{\partial(\Pi^m_{CC}-\Pi^m_{TC})}{\partial \rho}(\Pi^m_{TT}-\Pi^m_{CT})}{(\Pi^m_{CC}-\Pi^m_{TC}+\Pi^m_{TT}-\Pi^m_{CT})^2}.\end{aligned} \tag{2.21}$$

Combating with $\Pi^m_{CC}-\Pi^m_{TC}<0$ and $\Pi^m_{TT}-\Pi^m_{CT}<0$, we can easy know that $\frac{\partial x^*}{\partial \rho}>0$ if $\frac{\partial(\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho}<0$ and $\frac{\partial(\Pi^m_{CC}-\Pi^m_{TC})}{\partial \rho}>0$.

From $\Pi^m_{CC}-\Pi^m_{TC}<0$, we can deduce that $(\gamma^2-2\beta^2)c+\beta\gamma(c+\Delta c)+\alpha(2\beta+\gamma)+(\gamma-2\beta)(\mu_0+\rho\mu)-\alpha-\mu_0-(\gamma-\beta)(c+\Delta c)>0$.

Then we can derive the following equation:

$$\begin{aligned}\frac{\partial(\Pi^m_{CC}-\Pi^m_{TC})}{\partial \rho} &= \frac{-2\beta\mu(1-\theta-\Delta\theta)(2\beta-\gamma)[(\gamma^2-2\beta^2)c+\beta\gamma(c+\Delta c)]}{(2\beta+\gamma)^2(2\beta-\gamma)^2} \\ &+ \frac{-2\beta\mu(1-\theta-\Delta\theta)(2\beta-\gamma)[\alpha(2\beta+\gamma)+(\gamma-2\beta)(\mu_0+\rho\mu)]}{(2\beta+\gamma)^2(2\beta-\gamma)^2} \\ &> 0.\end{aligned}$$

Then we calculate the expression of $\frac{\partial(\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho}$, that is,

$$\begin{aligned}\frac{\partial(\Pi^m_{TT}-\Pi^m_{CT})}{\partial \rho} &= -\frac{\beta\mu(1-\theta-\Delta\theta)(2\beta+\gamma)^2[\alpha-\mu_0+(\gamma-\beta)c]^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2} \\ &-\frac{\beta\mu[(\gamma^2-2\beta^2)(c+\Delta c)+\beta\gamma c+\alpha(2\beta+\gamma)+(2\beta-\gamma)(\mu_0+\rho\mu)]}{(2\beta+\gamma)^2(2\beta-\gamma)^2}.\end{aligned} \tag{2.22}$$

From the above equation we can know that $Eq.(2.22)<0$ always holds if $(\gamma^2-2\beta^2)(c+\Delta c)+\beta\gamma c+\alpha(2\beta+\gamma)+(2\beta-\gamma)(\mu_0+\rho\mu)>0$. To prove it, $\Pi^m_{TT}-\Pi^m_{CT}<0$ is first simplified as follows,

$$\begin{aligned}&(\gamma^2-2\beta^2)(c+\Delta c)+\beta\gamma c+\alpha(2\beta+\gamma)+(2\beta-\gamma)(\mu_0+\rho\mu) \\ >\ &\frac{\sqrt{1-\theta-\Delta\theta}\sqrt{1-\rho\mu}[\alpha-\mu_0+(\gamma-\beta)c]}{\sqrt{1-\theta}} \\ >\ &0.\end{aligned} \tag{2.23}$$

According to the parameter constraint under Table 2.9, it holds that $\alpha-\mu_0+(\gamma-\beta)c > 0$, then we can get that $Eq.(2.23) > 0$ and $Eq.(2.22) < 0$.

Proof of Theorem 2.10

Proof. The proof of the existence and uniqueness of ρ can be translated into the monotonicity of the profit function U. In what follows, we discuss this problem in different situations since they can not occur simultaneously.

When $\Pi^m_{CC} - \Pi^m_{TC} < 0$, $\Pi^m_{TT} - \Pi^m_{CT} > 0$ and the CSR level holds that $x^* = 0$, the profit function U yields that

$$U = k_1\rho - k_3\rho^2. \tag{2.24}$$

If $\Pi^m_{CC} - \Pi^m_{TC} > 0$ and $\Pi^m_{TT} - \Pi^m_{CT} < 0$, we can obtain that $x^* = 1$. In this situation, it follows that

$$U = k_2 - k_3\rho^2. \tag{2.25}$$

The profit function U is a convex function of ρ in both cases above.

When $\Pi^m_{CC} - \Pi^m_{TC} < 0$ and $\Pi^m_{TT} - \Pi^m_{CT} < 0$, we show the monotonicity of profit function U. The first derivative of U with respect to ρ follows that

$$\frac{\partial U}{\partial \rho} = k_1(1 - x^*) + k_2\frac{\partial x^*}{\partial \rho} - (k_1\frac{\partial x^*}{\partial \rho} + 2k_3)\rho. \tag{2.26}$$

Let $\frac{\partial U}{\partial \rho} = 0$, we can get that

$$\rho = \frac{k_1(1 - x^*) + k_2\frac{\partial x^*}{\partial \rho}}{k_1\frac{\partial x^*}{\partial \rho} + 2k_3} := g(\rho). \tag{2.27}$$

From Theorem 1, it can be know that $\frac{\partial x^*}{\partial \rho} > 0$ and $x^* \in [0,1]$, which means $g(\rho) \geq 0$ and $g(\rho)$ is monotonically increasing. Therefore, we will only discuss the monotonicity of U in the following cases for $g(\rho) \in [0, \infty)$.

case 1. If $g(\rho) = 0$, it holds that $k_1 = k_2 = 0$, which contradicts the previous conclusion in Subsection 2.4.2. So we can obtain that $g(\rho) > 0$.

case 2. The parameters satisfy that $k_1(1-x^*) > (k_1-k_2)x^* + 2k_3$ when $g(\rho) > 1$. The condition is equivalent to that $\frac{\partial U}{\partial \rho} > 0$, so the profit function U monotonically increases when $\rho \in [0,1]$. In other words, the profit function U maximizes if and only if $\rho^* = 1$.

case 3. The parameters satisfy that $k_1(1-x^*) \leq (k_1-k_2)x^* + 2k_3$ when $g(\rho) \in (0,1]$. Due to the monotonicity of $g(\rho)$, there exists a unique $\rho^* \in (0,1]$ such that $\frac{\partial U}{\partial \rho} = 0$. That is, the profit function U is convex so that monotonically increases when $\rho \in (0, \rho^*)$ and decreases when $\rho \in (\rho^*, 1]$.

In summary, the profit function U is a monotone increasing function or a convex function in different situations. The existence and uniqueness of the optimal effort level ρ^* are proved.

The following table and figures reflect the changing rule of each payoff function in the media supervision mechanism under different parameters. In Table 2.7, $+$ means monotonically increasing and $-$ means monotonically decreasing with the independent variable.

Table 2.7: The change rule of each payoff concerning different parameters

Parameter	Π_{CC}	Π_{CT}	Π_{TC}	Π_{TT}	U
α	$+$	$+$	$+$	$+$	$+$
β	$-$	$-$	$-$	$-$	$-$
γ	$+$	$+$	$+$	$+$	$+$
c	$-$	$-$	$-$	$-$	$-$
k_1	\	\	\	\	$+$
k_2	\	\	\	\	$+$
k_3	\	\	\	\	$-$

The optimal values of each strategy profile in Table 2.1 are shown in the following table.

Table 2.8: Optimal value of each strategy profile.

	Strategy profile TT	Strategy profile TC/CT	Strategy profile CC
Price	$\frac{\alpha-\mu_0+\beta c}{2\beta-\gamma}$	$\frac{\alpha(2\beta+\gamma)-(2\beta-\gamma)\mu_0+2\beta^2 c+\beta\gamma(c+\Delta c)}{(2\beta+\gamma)(2\beta-\gamma)}$ $\frac{\alpha(2\beta+\gamma)+(2\beta-\gamma)\mu_0+2\beta^2(c+\Delta c)+\beta\gamma c}{(2\beta+\gamma)(2\beta-\gamma)}$	$\frac{\alpha+\mu_0+\beta(c+\Delta c)}{2\beta-\gamma}$
Demand	$\frac{\beta[\alpha-\mu_0+c(\gamma-\beta)]}{2\beta-\gamma}$	$\frac{\beta[\alpha(2\beta+\gamma)-(2\beta-\gamma)\mu_0+\beta\gamma(c+\Delta c)-(2\beta^2-\gamma^2)c]}{(2\beta+\gamma)(2\beta-\gamma)}$ $\frac{\beta[\alpha(2\beta+\gamma)+(2\beta-\gamma)\mu_0+\beta\gamma c-(2\beta^2-\gamma^2)(c+\Delta c)]}{(2\beta+\gamma)(2\beta-\gamma)}$	$\frac{\beta[\alpha+\mu_0+(c+\Delta c)(\gamma-\beta)]}{2\beta-\gamma}$
Payoff	$\frac{\beta[\alpha-\mu_0+c(\gamma-\beta)]^2}{(2\beta-\gamma)^2}$	$\frac{\beta[\alpha(2\beta+\gamma)-(2\beta-\gamma)\mu_0+\beta\gamma(c+\Delta c)-(2\beta^2-\gamma^2)c]^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2}$ $\frac{\beta[\alpha(2\beta+\gamma)+(2\beta-\gamma)\mu_0+\beta\gamma c-(2\beta^2-\gamma^2)(c+\Delta c)]^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2}$	$\frac{\beta[\alpha+\mu_0+(c+\Delta c)(\gamma-\beta)]^2}{(2\beta-\gamma)^2}$

Proof of Table 2.8. Taking the strategy profile CC as an example, we calculate the optimal value in Table 2.8. Inserting Eq. (2.4) into Eq. (2.5), the first derivative of Π_i^{CC} and Π_j^{CC} with respect to p_i and p_j yield that

$$\begin{cases} \frac{\partial \Pi_i^{CC}}{\partial p_i} = -2\beta p_i + \gamma p_j + \alpha + \mu_0 + \beta(c + \Delta c), \\ \frac{\partial \Pi_j^{CC}}{\partial p_j} = -2\beta p_j + \gamma p_i + \alpha + \mu_0 + \beta(c + \Delta c). \end{cases} \tag{2.28}$$

Let Eq. (2.28) equal to zero, we can calculate the optimal price of strategy profile CC:

$$p_i = p_j = \frac{\alpha + \mu_0 + \beta(c + \Delta c)}{2\beta - \gamma}. \tag{2.29}$$

Then the optimal values of profit and demand can be derived by Eq. (2.4) and Eq. (2.5).

For economical meaning, it is necessary for the demand $q_{S_i S_j}$ to be positive. Then we can calculate the parameters satisfy $\alpha > max\{(c + \Delta c)(\beta - \gamma) - \mu_0, c(\beta - \gamma) + \mu_0\}$ and $\alpha(2\beta + \gamma) > max\{(2\beta^2 - \gamma^2)(c + \Delta c) - (2\beta - \gamma)\mu_0 - \beta\gamma c, (2\beta^2 - \gamma^2)c + (2\beta - \gamma)\mu_0 - \beta\gamma(c + \Delta c)\}$. □

Under these conditions, the optimal payoffs in Table 2.8 have the following properties:

- As the increment of μ_0 and $\Delta\theta$, Π_{TT}^w decreases,
- Π_{TC}^w increases with respect to Δc, and decreases with respect to μ_0 and $\Delta\theta$,
- With the increase of μ_0 and Δc, Π_{CC}^w increases and decreases.

Proof. To prove these properties, we calculate the partial derivatives of the strategy payoff with respect to parameters μ_0 and Δc. For strategy

payoff Π_{TT}^{w}, we have

$$\frac{\partial \Pi_{TT}^{w}}{\partial \mu_0} = \frac{2\beta\left[\mu_0 - \alpha + c(\beta - \gamma)\right]}{(2\beta - \gamma)^2} < 0. \tag{2.30}$$

According to Eq. (2.30), it follows that the strategy payoff Π_{TT}^{w} is monotonically decreasing about μ_0.

The first derivative of Π_{TC}^{w} with respect to μ_0 and Δc yields that

$$\frac{\partial \Pi_{TC}^{w}}{\partial \mu_0} = \frac{2\beta(2\beta + \gamma)\left[\mu_0 - \alpha + c(\beta - \gamma)\right] - 2\beta^2\gamma\Delta c}{(2\beta - \gamma)^2(2\beta + \gamma)} < 0, \tag{2.31}$$

and

$$\frac{\partial \Pi_{TC}^{w}}{\partial \Delta c} = \frac{2\beta^3\gamma^2\Delta c + 2\beta^2\gamma(2\beta + \gamma)\left[\alpha - \mu_0 - c(\beta - \gamma)\right]}{(2\beta - \gamma)^2(2\beta + \gamma)^2}. \tag{2.32}$$

We can easily calculate that $Eq.(2.32) > 0$ when $\Delta c > 0$. The monotonicity of strategy payoff Π_{TC}^{w} is proved.

As for strategy payoff Π_{CC}^{w}, we can obtain the following conclusion if $0 < \mu_0 < 1$ and $\Delta c > 0$:

$$\begin{cases} \frac{\partial \Pi_{CC}^{w}}{\partial \mu_0} = \frac{2\beta[\mu_0 - \alpha + (c + \Delta c)(\beta - \gamma)]}{(2\beta - \gamma)^2} < 0, \\ \frac{\partial \Pi_{CC}^{w}}{\partial \Delta c} = \frac{2\beta(\gamma - \beta)\Delta c + 2\beta(\gamma - \beta)[\alpha + \mu_0 + c(\gamma - \beta)]}{(2\beta - \gamma)^2} < 0. \end{cases} \tag{2.33}$$

□

The optimal values of each strategy profile in Table 2.2 are shown in the following table.

The details in Tables 2.3-2.6 are shown as follows,

$$
\begin{aligned}
\Pi_{CC}^{hh} &= \frac{\beta(1-\theta)[\alpha+\mu_0+(c_h+\Delta c)(\gamma-\beta)]^2}{(2\beta-\gamma)^2},\\
\Pi_{CT}^{hh} &= \frac{\beta(1-\theta)(\alpha(2\beta+\gamma)+(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma c_h-(2\beta^2-\gamma^2)(c_h+\Delta c))^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{CC}^{ll} &= \frac{\beta(1-\theta)[\alpha+\mu_0+(c_l+\Delta c)(\gamma-\beta)]^2}{(2\beta-\gamma)^2},\\
\Pi_{CT}^{ll} &= \frac{\beta(1-\theta)(\alpha(2\beta+\gamma)+(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma c_l-(2\beta^2-\gamma^2)(c_l+\Delta c))^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{TT}^{hh} &= \frac{\beta(1-\theta-\Delta\theta)(1-\rho\mu)[\alpha-\mu_0+c_h(\gamma-\beta)]^2}{(2\beta-\gamma)^2},\\
\Pi_{TC}^{hh} &= \frac{\beta(1-\theta-\Delta\theta)(\alpha(2\beta+\gamma)-(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma(c_h+\Delta c)-(2\beta^2-\gamma^2)c_h)^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{TT}^{ll} &= \frac{\beta(1-\theta-\Delta\theta)(1-\rho\mu)[\alpha-\mu_0+c_l(\gamma-\beta)]^2}{(2\beta-\gamma)^2},
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{TC}^{ll} &= \frac{\beta(1-\theta-\Delta\theta)(\alpha(2\beta+\gamma)-(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma(c_l+\Delta c)-(2\beta^2-\gamma^2)c_l)^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{TT}^{hl} &= \frac{\beta(1-\theta-\Delta\theta)(1-\rho\mu)(-2\beta^2c_h+\beta\gamma c_l+\gamma^2c_h+(\alpha-\mu_0)(2\beta+\gamma))^2}{(2\beta-\gamma)^2},\\
\Pi_{TT}^{lh} &= \frac{\beta(1-\theta-\Delta\theta)(1-\rho\mu)(-2\beta^2c_l+\beta\gamma c_h+\gamma^2c_l+(\alpha-\mu_0)(2\beta+\gamma))^2}{(2\beta-\gamma)^2},\\
\Pi_{CC}^{hl} &= \frac{\beta(1-\theta)(-2\beta^2(c_h+\Delta c)+\beta\gamma(c_l+\Delta c)+\gamma^2(c_h+\Delta c)+(\alpha+\mu_0)(2\beta+\gamma))^2}{(2\beta-\gamma)^2},\\
\Pi_{CC}^{lh} &= \frac{\beta(1-\theta)(-2\beta^2(c_l+\Delta c)+\beta\gamma(c_h+\Delta c)+\gamma^2(c_l+\Delta c)+(\alpha+\mu_0)(2\beta+\gamma))^2}{(2\beta-\gamma)^2},\\
\Pi_{TC}^{hl} &= \frac{\beta(1-\theta-\Delta\theta)(2\beta(\alpha-\mu_0-\rho\mu)-2\beta^2c_h+\beta\gamma(c_l+\Delta c)+\gamma(\alpha+\mu_0+\rho\mu)+\gamma^2c_h)^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{TC}^{lh} &= \frac{\beta(1-\theta-\Delta\theta)(2\beta(\alpha-\mu_0-\rho\mu)-2\beta^2c_l+\beta\gamma(c_h+\Delta c)+\gamma(\alpha+\mu_0+\rho\mu)+\gamma^2c_l)^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{CT}^{hl} &= \frac{\beta(1-\theta)(2\beta(\alpha+\mu_0+\rho\mu)-2\beta^2(c_h+\Delta c)+\beta\gamma c_l+\gamma(\alpha-\mu_0-\rho\mu)+\gamma^2(c_h+\Delta c))^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2},\\
\Pi_{CT}^{lh} &= \frac{\beta(1-\theta)(2\beta(\alpha+\mu_0+\rho\mu)-2\beta^2(c_l+\Delta c)+\beta\gamma c_h+\gamma(\alpha-\mu_0-\rho\mu)+\gamma^2(c_l+\Delta c))^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2}.
\end{aligned}
$$

Table 2.9: Optimal value of each strategy profile under media supervision

	Strategy profile TT	Strategy profile TC/CT	Strategy profile CC
Price	$\frac{\alpha-\mu_0+\beta c}{2\beta-\gamma}$	$\frac{\alpha(2\beta+\gamma)-(2\beta-\gamma)(\rho\mu+\mu_0)+2\beta^2 c+\beta\gamma(c+\Delta c)}{(2\beta+\gamma)(2\beta-\gamma)}$ $\frac{\alpha(2\beta+\gamma)+(2\beta-\gamma)(\rho\mu+\mu_0)+2\beta^2(c+\Delta c)+\beta\gamma c}{(2\beta+\gamma)(2\beta-\gamma)}$	$\frac{\alpha+\mu_0+\beta(c+\Delta c)}{2\beta-\gamma}$
Demand	$\frac{\beta(1-\rho\mu)[\alpha-\mu_0+c(\gamma-\beta)]}{2\beta-\gamma}$	$\frac{\beta\left(\alpha(2\beta+\gamma)-(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma(c+\Delta c)-(2\beta^2-\gamma^2)c\right)}{(2\beta+\gamma)(2\beta-\gamma)}$ $\frac{\beta\left(\alpha(2\beta+\gamma)+(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma c-(2\beta^2-\gamma^2)(c+\Delta c)\right)}{(2\beta+\gamma)(2\beta-\gamma)}$	$\frac{\beta[\alpha+\mu_0+(c+\Delta c)(\gamma-\beta)]}{2\beta-\gamma}$
Payoff	$\frac{\beta(1-\rho\mu)[\alpha-\mu_0+c(\gamma-\beta)]^2}{(2\beta-\gamma)^2}$	$\frac{\beta(\alpha(2\beta+\gamma)-(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma(c+\Delta c)-(2\beta^2-\gamma^2)c)^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2}$ $\frac{\beta(\alpha(2\beta+\gamma)+(2\beta-\gamma)(\rho\mu+\mu_0)+\beta\gamma c-(2\beta^2-\gamma^2)(c+\Delta c))^2}{(2\beta+\gamma)^2(2\beta-\gamma)^2}$	$\frac{\beta[\alpha+\mu_0+(c+\Delta c)(\gamma-\beta)]^2}{(2\beta-\gamma)^2}$

3

Maximizing Social Welfare: Emission Allocation in Cournot Model

This Chapter is adapted from our working paper: Yanru Sun, Christopher Kops, Dries Vermeulen. "Maximizing Social Welfare: Emission Allocation in Cournot Model" (working paper).

Abstract

In the context of emission reduction in a Cournot duopoly model, we explore the government-optimal allocation of emission allowances to maximize social welfare. We model this situation as a two-stage Stackelberg game, where in stage one the government issues emission allowances to each firm, and in stage two both firms play a (restricted) Cournot game in which total carbon emission is not allowed exceed the allowance. This study has an intriguing conclusion: in many cases, the government-optimal strategy is either to enforce zero pollution or to allow firms to pollute freely. This reveals that standard procedures to allocate allowances, such as auctions and grandfathering, may be suboptimal, and thus in all likelihood an outcome of a political negotiation process rather than an autonomous welfare maximizing decision by a social planner.

3.1 Introduction

Due to global warming and environmental challenges, many countries have signed a series of climate agreements over the past 30 years, such as UNFCCC (1992), Kyoto Protocol (1997), and Paris Agreement (2015). Subsequently, many countries and governments have proposed the cap-and-trade system to control carbon emissions. For instance, the European Union (EU) Emissions Trading System covers more than 31 countries and controls approximately 45% of carbon emissions in Europe (Xiaoping Xu, W. Zhang, et al., 2017). China constructs a National Carbon Emissions Trading System and plans to reduce carbon emission intensity by 60–65% in 2030 based on the 2005 level.

In the cap-and-trade system, policymakers set a cap on emissions by allocating emission allowances to firms. After the initial allocation, firms can trade these emission allowances and subsequently produce under the constraint that carbon emissions should not exceed the total volume of allowances obtained. The initial allocation of allowances has been widely studied and is known to affect the outcome of emission reduction. Cramton and Kerr (2002) compared the effect of the auction and grandfather, which allocates emission allowances based on historical output. An auction is preferred to grandfathering because it can reduce tax distortions and provide greater incentives for innovation. Duro et al. (2020) used the claim problem to model the initial allocation of emission allowances. The results show that the constrained equal awards and α-minimal solutions are clearly equity-sensitive. Xiaoping Xu, M. Zhang, et al. (2022) analyzed grandfather-based allocation and benchmark-based allocation rules, and found that grandfathering generates less carbon emissions and more social welfare when the environmental damage coefficient is relatively high. Otherwise, Benchmarking leads to more carbon emissions and greater social well-being. Based on the data envelopment analysis model, Saen et al. (2024) presented a method for reallocating emission allowances and showed the efficacy of this model through a comprehensive case study.

Although methods for allocating emission allowances are widely stud-

ied, few studies have explored the goals that the government aims to achieve. For example, as environmental problems worsen, society is increasingly aware that pollution damages environmental welfare. In contrast, the government may not focus on pollution if economic development is urgent. Therefore, the government may focus on environmental welfare, economic welfare, or a balance between the two. This depends on the attitude of society that this government represents. Different attitudes towards environmental welfare may lead to different optimal choices regarding the allocation of emission allowances.

Given pollution emissions, we construct a Cournot model (Cournot, 1838) between two firms in the absence of regulation. This market consists of a "dirty" firm and a "clean" firm, in which the upper limit of the pollution level of the dirty firm is greater than that of the clean firm. Under perfect information, each firm simultaneously selects a quantity and a pollution level to maximize its profit. We prove that the optimal strategy for each firm is to adopt the upper limit of the pollution level in the Cournot equilibrium.

The government is endowed with a social welfare function that takes social damage into account. Social damage includes, for example, pollution, health costs, acid rain, global warming, and other environmental harms. To maximize social welfare, the government can cap social damage by allocating emission allowances to firms.

We analyze the maximization of social welfare and the corresponding optimal allocation through a Stackelberg game model (Von Stackelberg, 2010). In the first stage, the government decides on an allocation of emission allowances, which is the constraint on emissions of each firm. In the second stage, firms play a Cournot game with capacity constraints due to emission allowances. The first stage is then solved using backward induction.

In the Cournot equilibrium, the output and the pollution level are continuous as functions of the emission allowances. Therefore, in the first stage, the government can directly maximize social welfare through

the allocation of emission allowances. In equilibrium, the allocation of allowances by the government is not a continuous function of the damage parameter. Rather, the government chooses one out of a limited number of allowance allocations.

We also analyze a special case where the pollution damage is a linear function. When the differences in pollution levels and competition are both mild, the government-optimal strategy is either to enforce zero pollution or to allow firms to pollute freely. If the competition becomes intense, the government-optimal strategy includes another allocation in which only the emissions of the dirty firm are restricted.

The chapter is organized as follows. In Section 3.2, we analyze the Cournot model of two firms in the absence of regulation. Section 3.3 constructs a Stackelberg game and analyzes the Cournot equilibrium with emission allowance constraints. To maximize social welfare, Section 3.4 discusses the characteristics of government-optimal allocation in the general case. Section 3.5 explores the government-optimal allocation under linear pollution damage. Section 3.6 concludes the chapter. All proofs are collected in the Appendix.

3.2 Cournot model without regulation

Consider a duopoly Cournot model in which two firms $i = 1, 2$ choose their quantities simultaneously. For each $i = 1, 2$, let e_i denote the pollution level per unit of production for each firm. Thus, we implicitly assume that pollution is linear in production quantity. We assume that $0 \leq e_i \leq \alpha_i$, where α_i denotes the pollution level per unit of production for firm i when firm i does not take any action to reduce carbon emissions. Total pollution is $E_i = e_i \cdot q_i$ when firm i chooses production level q_i and per-unit pollution level e_i. Throughout this paper, we make the following assumption.

Assumption 3.1. $\alpha_1 \geq \alpha_2$.

Thus, we have one "dirty" firm 1, and a "clean" firm 2. We are interested in the effects of this asymmetry on government policy and the induced social welfare. We come back to this issue in the next section.

For each $i = 1, 2$, $C_i(q_i, e_i) = (c - e_i) \cdot q_i$ is the cost function of firm i when firm i produces an amount $q_i \geq 0$ and chooses pollution level $e_i \geq 0$. Thus, $c \geq 0$ denotes the marginal cost of production when firm i chooses to select pollution level $e_i = 0$. We assume that $c \geq \alpha_i$ for each $i = 1, 2$. When firm 1 produces an amount $q_1 \geq 0$ and firm 2 produces an amount $q_2 \geq 0$, the inverse demand is given by

$$p = a - q_1 - q_2 \qquad \text{where} \quad a > 0 \quad \text{is exogenously given.}$$

The profit for firm i is then given by

$$\pi_i(q_i, e_i, q_j) = p \cdot q_i - C(q_i, e_i) = (p - c + e_i) \cdot q_i.$$

It is clear that, absent regulation, a profit-maximizing firm i will set $e_i = \alpha_i$ regardless the production levels that are chosen.

Definition 3.2. *A strategy vector (q_1, e_1, q_2, e_2) is a Cournot equilibrium if for each firm $i = 1, 2$ it holds that $e_i = \alpha_i$ and*

$$\pi_i(q_i, e_i, q_j) \geq \pi_i(q_i', e_i', q_j) \quad \textit{for all} \quad q_i' \geq 0 \quad \textit{and} \quad e_i' \geq 0 \quad \textit{and} \quad j \neq i.$$

The next Proposition shows the Cournot equilibrium when the government does not regulate the market. In the Cournot equilibrium, neither firm actively reduces its per-unit pollution level, and the dirty firm has the higher production level and hence also the higher profit.

Proposition 3.3. *Without regulation, the (unique) Cournot equilibrium is given by $(q_1^c, e_1^c, q_2^c, e_2^c)$ where, for each $i = 1, 2$, it holds that*

$$q_i^c = \frac{a - c + 2\alpha_i - \alpha_j}{3} \quad \textit{and} \quad e_i^c = \alpha_i.$$

The corresponding price, profits and pollution are $p^c = \frac{a+2c-\alpha_i-\alpha_j}{3}$, $\pi_i^c = \left(\frac{a-c+2\alpha_i-\alpha_j}{3}\right)^2$ *and* $E_i^c = \alpha_i \cdot \frac{a-c+2\alpha_i-\alpha_j}{3}$.

Remark 3.4. *We have shown that, in the Cournot equilibrium,*

$$p^c = \frac{a + 2c - \alpha_1 - \alpha_2}{3}.$$

Thus, since in equilibrium price needs to exceed marginal cost, that is $p^c - c > 0$*, we assume the following in the remainder of this paper.*

Assumption 3.5. *We assume that* $\alpha_1 + \alpha_2 < a - c$.

3.3 Emission allowances

3.3.1 The Stackelberg framework

To control the pollution of firms, we introduce a new player, the government. Aiming for emissions reduction, the government sets a cap on emissions and allocates allowances. There are several methods to do this. For example, Cap and Trade sets a cap on total emissions and allocates the allowance to each firm freely, and firms are allowed to trade emission allowances. Command and Control directly imposes an emissions limit on each firm to reduce the total emissions.

Both of the above two methods set a fixed cap in each period, with the cap decreasing over time, and focus on how to implement emission reduction. Rather than considering the implementation of emissions reduction, we focus on what the government wants to achieve. Therefore, we use a social welfare function to measure the government's objective, which takes both economic and environmental performance into account. In particular, we assume that

$$W = \Pi + CS - D.$$

Here, given choices q_1, e_1, q_2, and e_2 by the firms, $\Pi = \pi_1 + \pi_2$ is the total profits of the firms,

$$CS = \frac{1}{2} \cdot (a - p)^2 = \frac{1}{2} \cdot (q_1 + q_2)^2$$

is the consumer surplus.

The pollution damage $D = D(E)$ is a function of total pollution $E = e_1 \cdot q_1 + e_2 \cdot q_2$. Given the fact that pollution causes damage to social welfare, we assume the following in the general case.

Assumption 3.6. *D is continuous, strictly increasing in total emission E. Moreover, $D(0) = 0$.*

This assumption implies that emissions always have a negative effect on social welfare. The assumption indicates that social welfare does not suffer any loss when emissions are absent or when pollution damage is ignored.

As shown in Fig. 3.1 below, we view the interaction between government and competing firms as a two-stage Stackelberg game, in which the government is the leader and firms are followers. The timing is as follows.

Stage 1. With the goal of maximizing social welfare function

$$W = \Pi + CS - D,$$

the government decides on an allocation (A_1, A_2) of emission allowances. To ensure the reduction of emissions, we assume that $A_i \leq E_i^c$ for each $i = 1, 2$.

Stage 2. Given the emission allowances (A_1, A_2), firms engage in Cournot competition, choosing the variables e_i and q_i, where e_i denotes the pollution level and q_i is the quantity, in such a way that

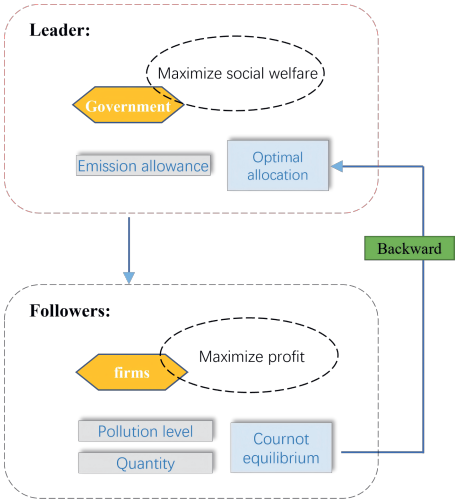


Figure 3.1: The illustration of the interaction between the government and firms.

$e_i \cdot q_i \leq A_i$. In this stage, each firm is assumed to have perfect information, knowing A_1, A_2, α_1, and α_2.

We use backwards induction to analyze the resulting Stackelberg game.

3.3.2 Cournot equilibrium under fixed allocation

Thus, we first analyze Stage 2 of the Stackelberg game, taking the allocation (A_1, A_2) of allowances by the government as given. To maximize profit, each firm $i \in \{1, 2\}$ chooses an quantity-pollution pair (q_i, e_i) such that $0 \leq e_i \leq \alpha_i$ and $e_i \cdot q_i \leq A_i$. These choices are made simultaneously. Note that the choice e_j of firm j does not directly affect the profit of firm i. Thus, in order to compute the Cournot equilibrium, we first compute best responses of firm i against choice q_j of firm $j \neq i$.

Lemma 3.7. *Suppose that $q_j \geq 0$ is given. Then the best response (q_i, e_i) of firm $i \neq j$ is*

$$BR_i(q_j) = \begin{cases} \left(\frac{1}{2} \cdot (a - c - q_j + \alpha_i), \alpha_i\right) & \text{if } q_j > \alpha_i + B_i \\ \left(\frac{A_i}{\alpha_i}, \alpha_i\right) & \text{if } B_i \leq q_j \leq \alpha_i + B_i \\ \left(\frac{1}{2} \cdot (a - c - q_j), 2A_i \cdot (a - c - q_j)^{-1}\right) & \text{if } q_j < B_i \end{cases}$$

where $B_i = a - c - \frac{2A_i}{\alpha_i}$. Consequently, $BR_i(q_j)$ is a continuous function.

Remark 3.8. *The equality $e_i = \alpha_i$ holds in the first two cases. In the first case of $q_j > \alpha_i + B_i$, the best response takes the same form as in the Cournot model (see Proposition 3.3). In the second and third cases, $\alpha_i \cdot q_i = A_i$ is binding. Note that in the second case of $B_i \leq q_j \leq \alpha_i + B_i$, each of the constraints is binding and holds with equality.*

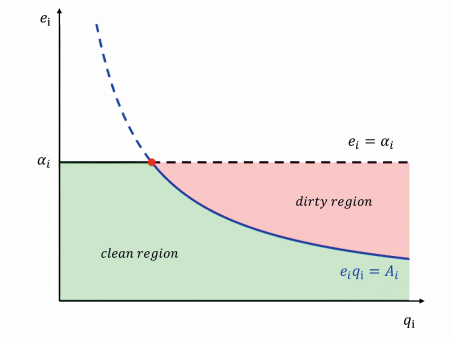


Figure 3.2: The feasible region for firm i.

Remark 3.8 implies that the best response must occur on the boundaries. As shown in Fig. 3.2, the first case is indicated by the black boundary, the second case corresponds to the red point, and the third case is reflected by the blue line.

Theorem 3.9. *Let $(q_1^*, e_1^*, q_2^*, e_2^*)$ be an equilibrium. Then $e_1^* \cdot q_1^* = A_1$ and $e_2^* \cdot q_2^* = A_2$.*

By Theorem 3.9 and Lemma 3.7, in any equilibrium (q_1, e_1, q_2, e_2) it holds that $q_1 \leq \alpha_2 + B_2$ and $q_2 \leq \alpha_1 + B_1$. We discuss the 4 remaining possibilities in Lemma 3.7.

Theorem 3.10. *Let the allocation $(A_1, A_2) \in [0, E_1^c] \times [0, E_2^c]$ be given. Then there is a unique Cournot equilibrium, which is given by the following description.*

(1) *If $2\alpha_j A_i + \alpha_i A_j \geq \alpha_i \cdot \alpha_j \cdot (a - c)$, for each $i = 1, 2$ and $j \neq i$. Then the equilibrium is*

$$(q_1^*, e_1^*, q_2^*, e_2^*) = \left(\frac{A_1}{\alpha_1}, \alpha_1, \frac{A_2}{\alpha_2}, \alpha_2 \right).$$

(2) *For fixed $i = 1, 2$ and $j \neq i$. Suppose that $2\alpha_j A_i + \alpha_i A_j < \alpha_i \cdot \alpha_j \cdot (a - c)$ and $A_i \geq \frac{\alpha_i}{3}(a - c)$. Then the equilibrium $(q_i^*, e_i^*, q_j^*, e_j^*)$ is given by*

$$\begin{cases} (q_i^*, e_i^*) = \left(\frac{A_i}{\alpha_i}, \alpha_i \right), \\ (q_j^*, e_j^*) = \left(\frac{1}{2} \cdot \left(a - c - \frac{A_i}{\alpha_i} \right), 2A_j \cdot \left(a - c - \frac{A_i}{\alpha_i} \right)^{-1} \right). \end{cases}$$

(3) *If $A_i < \frac{\alpha_i}{3}(a - c)$, for $i = 1, 2$. Then the equilibrium $(q_1^*, e_1^*, q_2^*, e_2^*)$ is given by*

$$(q_1^*, e_1^*) = (q_2^*, e_2^*) = \left(\frac{1}{3} \cdot (a - c), 3A_i \cdot (a - c)^{-1} \right).$$

Remark 3.11. *The regions shown in Fig. 3.3 correspond to the four forms of equilibrium in Theorem 3.10. Formally,*

$$\begin{aligned} R(1) &= \{(A_1, A_2) \mid 2\alpha_j A_i + \alpha_i A_j \geq \alpha_i \cdot \alpha_j \cdot (a - c), \quad i = 1, 2 \text{ and } j \neq i\} \\ R(2) &= \left\{(A_1, A_2) \mid 2\alpha_j A_i + \alpha_i A_j < \alpha_i \cdot \alpha_j \cdot (a - c) \text{ and } A_i \geq \frac{\alpha_i}{3}(a - c)\right\} \\ R(3) &= \left\{(A_1, A_2) \mid A_i < \frac{\alpha_i}{3}(a - c) \quad \text{for } i = 1, 2\right\}. \end{aligned}$$

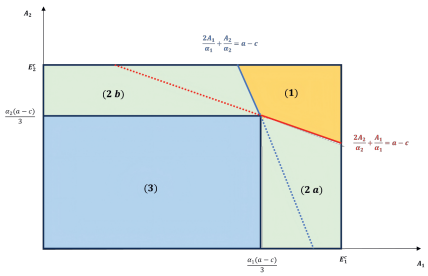


Figure 3.3: The regions of the Cournot equilibrium under allocation (A_1, A_2).

The region $R(2)$ is subdivided in $R(2a)$ and $R(2b)$, where $R(2a)$ corresponds to the choice $i = 1$ and $j = 2$, and $R(2b)$ corresponds to the choice $i = 2$ and $j = 1$.

Note that the equilibrium function, the function that assigns the Cournot equilibrium $(q_1^*, e_1^*, q_2^*, e_2^*)$ for each allocation (A_1, A_2) of emission allowances, is continuous, as the different forms of equilibrium coincide on the boundaries.

Remark 3.12. *Theorem 3.10 shows that the 4 regions have no overlapping parts. Furthermore, the different forms of equilibrium coincide at the boundaries. Thus, the equilibrium function is continuous.*

3.3.3 The general form of the social welfare function

Next we analyze Stage 1 of the Stackelberg game. Recall that, when the firms choose strategies (q_1, e_1) and (q_2, e_2), the social welfare function is given by

$$W = \Pi + CS - D.$$

Under the equilibrium in Stage 2, total firm profit is $\Pi = \pi_1 + \pi_2 = p \cdot (q_1^* + q_2^*)$, consumer surplus is $CS = \frac{1}{2}(q_1^* + q_2^*)^2$, and total carbon

emission is given by $D = D(E^*)$, where $E^* = e_1^* \cdot q_1^* + e_2^* \cdot q_2^*$. Based on Theorem 3.9, we have $D = D(A_1 + A_2)$. Thus the social welfare function is

$$\begin{aligned} W &= p(q_1^* + q_2^*) + \frac{1}{2}(q_1^* + q_2^*)^2 - D(A_1 + A_2) \\ &= (a - c - q_1^* - q_2^*)(q_1^* + q_2^*) + \frac{1}{2}(q_1^* + q_2^*)^2 - D(A_1 + A_2) \\ &= (a - c)(q_1^* + q_2^*) - \frac{1}{2}(q_1^* + q_2^*)^2 - D(A_1 + A_2). \end{aligned}$$

In order to maximize social welfare, the government acts as a leader and chooses emission allowances (A_1, A_2), which determine the equilibrium strategy (q_i^*, e_i^*) of each firm. Using Theorem 3.10, we can express this function as $W = W(A_1, A_2)$. The result is presented in the Theorem below.

Theorem 3.13. *Let (A_1, A_2) be the emission allowance strategy chosen by the government. Then the social welfare in the Stackelberg game is given by*

$$W(A_1, A_2) = \begin{cases} (a-c)C_0 - \frac{1}{2}C_0^2 - D & \text{if } (A_1, A_2) \in R(1) \\ \frac{1}{2}(a-c)C_1 - \frac{1}{8}C_1^2 - D & \text{if } (A_1, A_2) \in R(2a) \\ \frac{1}{2}(a-c)C_2 - \frac{1}{8}C_2^2 - D & \text{if } (A_1, A_2) \in R(2b) \\ \frac{4}{9}(a-c)^2 - D & \text{if } (A_1, A_2) \in R(3), \end{cases}$$

where $C_0 = \frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2}$, and $C_i = a - c + \frac{A_i}{\alpha_i}$ for each $i = 1, 2$, and $D = D(A_1 + A_2)$.

The government wishes to maximize the resulting welfare function, using the emission allowances (A_1, A_2) as the policy tool. In the next section, we analyze the characteristics of the government-optimal allocation in the general case.

3.4 The government-optimal allocation in the general case

A precise definition for a government-optimal allocation is given below.

Definition 3.14. *Let $d \geq 0$ be fixed. An allocation (A_1^*, A_2^*) of allowances is called government-optimal if*

$$W(A_1^*, A_2^*) \geq W(A_1, A_2) \quad \textit{for all } A_i \in [0, E_i^c], i = 1, 2.$$

Recalling Assumption 3.6, D is continuous, strictly increasing in total emission E, and $D(0) = 0$. We identify candidates for the government-optimal allocation, which achieve the maximum social welfare within each range.

Proposition 3.15. *Let (A_1, A_2) be a government-optimal allocation of allowances. If $(A_1, A_2) \in R(3)$, then $(A_1, A_2) = (0, 0)$ and the maximum social welfare is $W = \frac{4}{9}(a-c)^2$.*

Proposition 3.16. *Let (A_1, A_2) be a government-optimal allocation of allowances. If $(A_1, A_2) \in R(2a)$, then $A_2 = 0$ and $A_1 \in [\frac{\alpha_1(a-c)}{3}, E_1^c]$.*

Remark 3.17. *Due to the symmetry, a government-optimal allocation of allowances $(A_1, A_2) \in R(2b)$ satisfies that $A_1 = 0$ and $A_2 \in [\frac{\alpha_2(a-c)}{3}, E_2^c]$.*

The above results demonstrate some properties of $R(2)$ and $R(3)$ in the general case. Through Proposition 3.15, if (A_1, A_2) belongs to $R(3)$, then there exists a unique allocation $(0, 0)$ that maximizes social welfare. In this case, the optimal strategy for the government is to implement a strict policy that limits any pollution from firms.

3.5 The government-optimal allocation under linear pollution damage

In this section, we consider a special case where the pollution damage is linear, which is

$$D = d \cdot (E_1 + E_2), \qquad \text{where } d \geq 0.$$

The government wishes to maximize the resulting welfare function, using the emission allowances (A_1, A_2) as the policy tool. In the remainder of this Chapter, we analyze the case where firms only have mild differentiation in emission, $\alpha_1 \leq 2 \cdot \alpha_2$.

Define

$$d_0 = \frac{2(a-c)(\alpha_1+\alpha_2) - (\alpha_1+\alpha_2)^2}{18 \cdot (E_1^c + E_2^c)},$$

and

$$T = \frac{5\alpha_1\alpha_2 - \alpha_2^2 + \sqrt{6\alpha_1^4 + 2\alpha_1^3\alpha_2 + 19\alpha_1^2\alpha_2^2 - 4\alpha_1\alpha_2^3 + 9\alpha_2^4}}{2(\alpha_1+\alpha_2)}.$$

If $a - c \geq T$, the competition is relatively moderate due to substantial potential demand. The following Theorem shows the government-optimal allocation when firms are mildly differentiated, and the competition is relatively moderate.

Theorem 3.18. *Suppose that firms are mildly differentiated. Suppose further that $a - c \geq T$. Then the government-optimal allocation is*

$$GOA(d) = \begin{cases} (E_1^c, E_2^c) & \text{if } 0 \leq d \leq d_0 \\ (0,0) & \text{if } d \geq d_0. \end{cases}$$

As shown in Fig. 3.4, the government-optimal allocation consists of two extreme allocations when firms are mildly differentiated and the

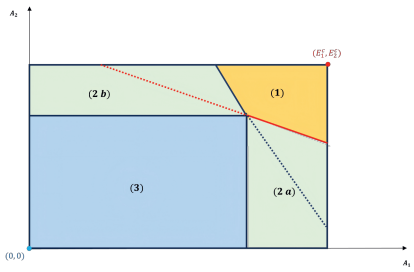


Figure 3.4: The government-optimal allocation under **C1** and **C2**.

competition is relatively moderate. The government faces a binary choice: either mandate clean production for all firms, or impose no pollution restrictions on any firm.

Write $B = (2\alpha_1 + \alpha_2) \cdot (a - c) + (\alpha_1 - 2\alpha_2) \cdot (\alpha_1 - \alpha_2)$. Define

$$d_1 = \frac{B - \sqrt{B^2 - \alpha_1^2(a-c)^2}}{3\alpha_1^2}.$$

Theorem 3.19. *Suppose that firms are mildly differentiated. Suppose further that $a - c < T$. Then the government-optimal allocation is*

$$GOA(d) = \begin{cases} (E_1^c, E_2^c) & \text{if} \quad 0 \le d \le \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \\ (A^1, E_2^c) & \text{if} \quad \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} < d < d_1 \\ (0,0) & \text{if} \quad d > d_1. \end{cases}$$

where $A^1 = \alpha_1 \cdot \left(\frac{2(a-c)-2\alpha_2+\alpha_1}{3} - \alpha_1 \cdot d \right)$. If $d = d_1$, then both (A^1, E_2^c) and $(0,0)$ are government-optimal.

Compared with Theorem 3.18, the government-optimal allocation includes an additional intermediate allocation (A^1, E_2^c) if the competition is relatively intense. Specifically, the government only needs to restrict the emissions of the dirty firm to a certain extent, while leaving the clean firm unregulated.

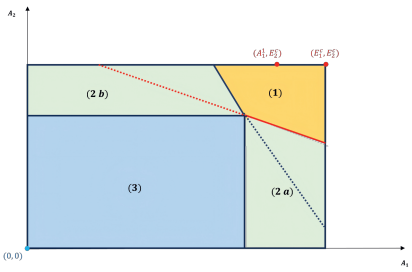


Figure 3.5: The government-optimal allocation under **C1** and ¬**C2**.

Remark 3.20. *If D is convex, increasing and sufficiently steep, then the result in Theorem 3.18 remains true. That is, the government-optimal allocation is either to enforce zero pollution or to allow firms to pollute freely when the damage function is convex and satisfies certain conditions.*

3.6 Conclusion

In this chapter, we explore the optimal allocation of allowances to maximize social welfare. We constructed a Stackelberg game in which the government moves first to allocate the emission allowances, and then firms play a Cournot game with emission allowance constraints. For the government, social welfare includes not only producer surplus and consumer surplus but also social damage resulting from emissions. For firms, we introduce an additional decision regarding the pollution level in the restricted Cournot competition.

Using backward induction, we show that the output and the pollution level are continuous as functions of the emission allowances in the Cournot equilibrium. Therefore, the government can directly maximize social welfare through the allocation of emission allowances. We analyze the candidates of government-optimal allocations when the pollution damage function is in a general form or linear. We provide a counterintuitive conclusion: when competition and the difference in pollution level are mild, the government-optimal strategy is either to

enforce zero pollution or to allow firms to pollute freely. Therefore, auctions, grandfathering, or other standard procedures may be suboptimal. This reveals that the cap-and-trade system may be the result of political negotiation rather than a means to maximize social welfare.

3.7 Appendix

Proof of Proposition 3.3. Fix $i = 1, 2$. Clearly $e_i^c = \alpha_i$. The profit maximization problem for firm i simplifies to

$$\max_{q_i} \pi_i = q_i \cdot (a - q_i - q_j - c + \alpha_i).$$

Since the objective function is a concave parabola, the maximum location is

$$q_i = \frac{a - q_j - c + \alpha_i}{2}.$$

So, in the Cournot equilibrium it holds that

$$\begin{aligned} 2 \cdot q_1 + q_2 &= a - c + \alpha_1 \\ 2 \cdot q_2 + q_1 &= a - c + \alpha_2. \end{aligned}$$

So, $q_1 - q_2 = \alpha_1 - \alpha_2$. It follows that

$$q_1^c = \frac{a - c + 2 \cdot \alpha_1 - \alpha_2}{3} \quad \text{and} \quad q_2^c = \frac{a - c + 2 \cdot \alpha_2 - \alpha_1}{3}.$$

Now p^c, π_i^c, and E_i^c can be computed directly from their respective definitions. □

Proof of Lemma 3.7. Let the production quantity q_j of firm j be given. A best response of firm i against q_j solves the profit maximization problem

$$\begin{aligned} \max_{e_i, q_i} \quad & \pi_i = (a - q_i - q_j - c) \cdot q_i + e_i \cdot q_i \\ \text{subject to} \quad & 0 \leq e_i \leq \alpha_i \text{ and } e_i \cdot q_i \leq E_i. \end{aligned}$$

For any (q_i, e_i) in the feasible region with $e_i < \alpha_i$ and $e_i \cdot q_i < E_i$,

the choice $(q_i, e_i + \epsilon)$ is strictly better than (q_i, e_i) for $\epsilon > 0$ sufficiently small.

Let (q_i, e_i) be a maximum location. Then, by the previous observation, at least one of the two equalities $e_i q_i = E_i$ or $e_i = \alpha_i$ must hold. We consider three cases.

A. If $q_j > \alpha_i + B_i$. Define $q_i^* = \frac{a - q_j - c + \alpha_i}{2}$. First, note that

$$q_j > \alpha_i + B_i = \alpha_i + a - c - \frac{2A_i}{\alpha_i},$$

so that $q_i^* < \frac{A_i}{\alpha_i}$. Then the strategy (q_i^*, α_i) is in the feasible region of firm i. However, the strategy (q_i^*, α_i) is the maximum location of the optimization problem

$$\begin{aligned} \max_{q_i, e_i} \quad & \pi_i = (a - q_i - q_j - c + e_i) \cdot q_i \\ \text{subject to} \quad & e_i \leq \alpha_i. \end{aligned}$$

The above problem has a larger feasible region than the original maximization problem. Thus, (q_i^*, α_i) is also the maximum location of the maximization problem with the more restricted region.

B. If $q_j < B_i$. Define $q_i^{**} = \frac{a - q_j - c}{2}$ and $e_i^{**} = \frac{A_i}{q_i^{**}}$. Since $q_j < B_i$, it follows that

$$q_j < B_i = a - c - \frac{2A_i}{\alpha_i}, \quad \text{so that} \quad q_i^{**} = \frac{a - q_j - c}{2} > \frac{A_i}{\alpha_i}.$$

So, $e_i^{**} = \frac{A_i}{q_i^{**}} < \alpha_i$. It follows that (q_i^{**}, e_i^{**}) is in the feasible region. Moreover, (q_i^{**}, e_i^{**}) is the maximum location of the optimization problem

$$\begin{aligned} \max_{e_i \geq 0, q_i \geq 0} \quad & \pi_i = (a - q_i - q_j - c) \cdot q_i + e_i \cdot q_i \\ \text{subject to} \quad & e_i \cdot q_i \leq A_i. \end{aligned}$$

Since this problem has a larger feasible region than the original problem, (q_i^{**}, e_i^{**}) is also the maximum location of the original optimization problem.

C. If $B_i \leq q_j \leq \alpha_i + B_i$. Note that from the observations in A and B it follows that for both larger domains, the optimal choice is outside the feasible region. Thus, in this case the point where both inequalities are binding, $e_i = \alpha_i$ and $e_i \cdot q_i = A_i$, is the maximum location. This is the point $(q_i, e_i) = (\frac{A_i}{\alpha_i}, \alpha_i)$. □

Proof of Theorem 3.9. Let $(q_1^*, e_1^*, q_2^*, e_2^*)$ be an equilibrium. The proof is in two steps.

A. Claim. Suppose that $q_j^* > \alpha_i + B_i$. Then

$$\begin{cases} q_j^* > \frac{1}{3} \cdot (a - c + 2\alpha_j - \alpha_i) \\ q_i^* < \frac{1}{3} \cdot (a - c + 2\alpha_i - \alpha_j). \end{cases}$$

In particular, $q_i^* \leq \alpha_j + B_j$.

Proof of Claim. Assume that $q_j^* > \alpha_i + B_i$. Since $A_i \leq E_i^c = \frac{1}{3} \cdot \alpha_i \cdot (a - c + 2\alpha_i - \alpha_j)$, we have that

$$\begin{aligned} q_j^* > \alpha_i + B_i &= \alpha_i + a - c - \frac{2A_i}{\alpha_i} \geq \alpha_i + a - c - \frac{2E_i^c}{\alpha_i} \\ &= \alpha_i + a - c - \frac{2}{3}(a - c + 2\alpha_i - \alpha_j) \\ &= \frac{1}{3}(a - c + 2\alpha_j - \alpha_i). \end{aligned}$$

Further, using Lemma 3.7 in the first step, we have that

$$\begin{aligned} q_i^* &= \frac{1}{2} \cdot (a - c - q_j^* + \alpha_i) = \frac{1}{2} \cdot (a - c + \alpha_1) - \frac{1}{2} \cdot q_j^* \\ &< \frac{1}{2} \cdot (a - c + \alpha_i) - \frac{1}{6} \cdot (a - c + 2\alpha_j - \alpha_i) \\ &= \frac{1}{3} \cdot (a - c + 2\alpha_i - \alpha_j). \end{aligned}$$

End proof of Claim.

B. It suffices to show that $e_1 \cdot q_1^* = A_1$ for firm 1. Suppose by contradiction that this equality does not hold. Then, by Lemma 3.7 we know that $q_2^* > \alpha_1 + B_1$. Then, by the above Claim, we know that $q_1^* \leq \alpha_2 + B_2$. We consider two cases.

Case 1. If $B_2 \leq q_1^* \leq \alpha_2 + B_2$. Then $q_1^* = \frac{1}{2} \cdot (a - c - q_2^* + \alpha_1)$ and $q_2^* = \frac{A_2}{\alpha_2}$ by Lemma 3.7. So,

$$\begin{aligned} a - c - q_2^* + \alpha_1 = 2 \cdot q_1^* &\leq 2\alpha_2 + 2B_2 \\ &= 2\alpha_2 + 2(a - c) - \frac{4A_2}{\alpha_2} \\ &= 2\alpha_2 + 2(a - c) - 4q_2^*. \end{aligned}$$

The inequality

$$a - c - q_2^* + \alpha_1 \leq 2\alpha_2 + 2(a - c) - 4q_2^*$$

can be rewritten to $q_2^* \leq \frac{1}{3} \cdot (a - c + 2\alpha_2 - \alpha_1)$. This contradicts the above Claim.

Case 2. If $q_1^* \leq B_2$. Then $q_2^* = \frac{1}{2} \cdot (a - c - q_1^*)$ and $q_1^* = \frac{1}{2} \cdot (a - c - q_2^* + \alpha_1)$ by Lemma 3.7. Solving for q_2^* yields

$$q_2^* = \frac{1}{3} \cdot (a - c - \alpha_1).$$

Since $\alpha_2 \geq 0$, it follows that $q_2^* \leq \frac{1}{3} \cdot (a - c - \alpha_1 + 2\alpha_2)$. This contradicts the above Claim. □

Lemma 3.21. *Let $A_1 \leq E_1^c$ and $A_2 \leq E_2^c$. Then $\frac{A_2}{\alpha_2} \leq \alpha_1 + B_1$ and $\frac{A_1}{\alpha_1} \leq \alpha_2 + B_2$.*

Proof of Lemma 3.21. Since $B_1 = a - c - \frac{2A_1}{\alpha_1}$, $E_2^c = \frac{\alpha_1}{3}(a - c + 2\alpha_2 - \alpha_1)$, and $E_1^c = \frac{\alpha_1}{3}(a - c + 2\alpha_1 - \alpha_2)$, it follows that

$$\begin{aligned}
\frac{A_2}{\alpha_2} \leq \frac{E_2^c}{\alpha_2} &= \frac{1}{3} \cdot (a - c + 2\alpha_2 - \alpha_1) \\
&= \alpha_1 + a - c - \frac{2}{3} \cdot (a - c + 2\alpha_1 - \alpha_2) \\
&= \alpha_1 + a - c - \frac{2E_1^c}{\alpha_1} \\
&\leq \alpha_1 + a - c - \frac{2A_1}{\alpha_1} \\
&= \alpha_1 + B_1.
\end{aligned}$$

A similar argument shows that $\frac{A_1}{\alpha_1} \leq \alpha_2 + B_2$. □

Proof of Theorem 3.10. Let (A_1, A_2) with $0 \leq A_1 \leq E_1^c$ and $0 \leq A_2 \leq E_2^c$ be the allocation of allowances. We show that there is a unique Cournot equilibrium that is given by the description above. Let $(q_1^*, e_1^*, q_2^*, e_2^*)$ be an equilibrium for the allocation (A_1, A_2). By Lemma 3.7 and Theorem 3.9, we need to consider four cases, where case B below in fact covers two asymmetric cases.

A. If $B_1 \leq q_2^* \leq \alpha_1 + B_1$ and $B_2 \leq q_1^* \leq \alpha_2 + B_2$. Then by Lemma 3.7 we have that

$$(q_1^*, e_1^*) = \left(\frac{A_1}{\alpha_1}, \alpha_1\right) \qquad \text{and} \qquad (q_2^*, e_2) = \left(\frac{A_2}{\alpha_2}, \alpha_2\right).$$

B. If $q_2^* < B_1$ and $B_2 \leq q_1^* \leq \alpha_2 + B_2$. Then by Lemma 3.7 we have

that

$$\begin{cases} (q_1^*, e_1^*) = \left(\frac{1}{2} \cdot \left(a - c - \frac{A_2}{\alpha_2} \right), 2A_1 \cdot \left(a - c - \frac{A_2}{\alpha_2} \right)^{-1} \right), \\ (q_2^*, e_2) = \left(\frac{A_2}{\alpha_2}, \alpha_2 \right). \end{cases}$$

C. If $q_2^* < B_1$ and $q_1^* < B_2$. Then by Lemma 3.7 we have that

$$(q_1^*, e_1^*) = (q_2^*, e_2^*) = \left(\frac{1}{3} \cdot (a - c), 3A_i \cdot (a - c)^{-1} \right).$$

Thus, only four types of Cournot equilibrium exist, type A, type B (which has two asymmetric subcases), and type C. We show that the allowances (A_1, A_2) for which each type occurs partition the set of allowances into four disjoint regions.

I. Note that the strategy profile of type A is a Cournot equilibrium precisely when

$$B_1 \leq \frac{A_2}{\alpha_2} \leq \alpha_1 + B_1 \quad \text{and} \quad B_2 \leq \frac{A_1}{\alpha_1} \leq \alpha_2 + B_2.$$

By Lemma 3.21 we know that $\frac{A_i}{\alpha_i} \leq \alpha_j + B_j$. So, the above inequalities are equivalent to

$$B_1 \leq \frac{A_2}{\alpha_2} \quad \text{and} \quad B_2 \leq \frac{A_1}{\alpha_1}.$$

So, since $B_i = a - c - \frac{2A_i}{\alpha_i}$, we have that $B_i \leq \frac{A_j}{\alpha_j}$ is equivalent to

$$\alpha_i \cdot \alpha_j \cdot (a - c) - 2\alpha_j \cdot A_i = \alpha_i \cdot \alpha_j \cdot B_i \leq \alpha_i \cdot A_j.$$

This proves case (1) of the above statement.

II. The strategy profile of type B is a Cournot equilibrium precisely

when

$$\frac{A_2}{\alpha_2} < B_1 \quad \text{and} \quad B_2 \leq \frac{1}{2} \cdot \left(a - c - \frac{A_2}{\alpha_2} \right) \leq \alpha_2 + B_2.$$

Since $B_2 = a - c - \frac{2A_2}{\alpha_2}$, this is equivalent to

$$\frac{A_2}{\alpha_2} < B_1 \quad \text{and} \quad B_2 \leq \frac{1}{2} \cdot \left(B_2 + \frac{A_2}{\alpha_2} \right) \leq \alpha_2 + B_2.$$

Which is clearly equivalent to

$$\frac{A_2}{\alpha_2} < B_1 \quad \text{and} \quad B_2 \leq \frac{A_2}{\alpha_2} \leq 2\alpha_2 + B_2.$$

Since $\frac{E_2^c}{\alpha_2} \leq \frac{a-c+2\alpha_1}{3}$, this is then equivalent to

$$\frac{A_2}{\alpha_2} < B_1 \quad \text{and} \quad B_2 \leq \frac{A_2}{\alpha_2}.$$

Since $B_1 = a - c - \frac{2A_1}{\alpha_1}$, the inequality $\frac{A_2}{\alpha_2} < B_1$ is equivalent to

$$2 \cdot \alpha_2 A_1 + \alpha_1 \cdot A_2 < \alpha_2 \cdot \alpha_1 \cdot (a - c).$$

Since $B_2 = a - c - \frac{2A_2}{\alpha_2}$, the inequality $B_2 \leq \frac{A_2}{\alpha_2}$ is equivalent to

$$A_2 \geq \frac{\alpha_2 \cdot (a - c)}{3}.$$

This proves case (2) of the statement.

III. The strategy profile of type B is a Cournot equilibrium precisely when

$$\frac{1}{3} \cdot (a - c) < B_1 \quad \text{and} \quad \frac{1}{3} \cdot (a - c) < B_2.$$

Since $B_1 = a - c - \frac{2A_1}{\alpha_1}$, the inequality $\frac{1}{3} \cdot (a-c) < B_1$ is equivalent to

$$\frac{A_1}{\alpha_1} < \frac{a-c}{3}.$$

A similar argument holds for the other inequality. This proves case (3) of the above statement. □

Proof of Theorem 3.13. If $(A_1, A_2) \in R(1)$. Then by Theorem 3.10 the constrained Cournot equilibrium strategies are $e_i^* = \alpha_i$ and $q_i^* = \frac{A_i}{\alpha_i}$ for each $i = 1, 2$. Then the social welfare function is:

$$W = (a-c)(\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2}) - \frac{1}{2}(\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2})^2 - D(A_1 + A_2).$$

If $(A_1, A_2) \in R(2a)$. Then by Theorem 3.10 the constrained Cournot equilibrium strategies are $e_1^* = \alpha_1$, $q_1^* = \frac{A_1}{\alpha_1}$ and $q_2^* = \frac{a-c-q_1^*}{2}$ $e_2^* = \frac{A_2}{q_2^*}$. Then the social welfare is

$$\begin{aligned} W &= (a-c) \cdot (q_1^* + q_2^*) - \frac{1}{2}(q_1^* + q_2^*)^2 - D(A_1 + A_2), \\ &= (a-c) \cdot (\frac{A_1}{\alpha_1} + \frac{a - c - \frac{A_1}{\alpha_1}}{2}) - \frac{1}{2}(\frac{A_1}{\alpha_1} + \frac{a - c - \frac{A_1}{\alpha_1}}{2})^2 \\ &\quad -D(A_1 + A_2), \\ &= \frac{1}{2}(a-c)(a - c + \frac{A_1}{\alpha_1}) - \frac{1}{8}(a - c + \frac{A_1}{\alpha_1})^2 - D(A_1 + A_2). \end{aligned}$$

If $(A_1, A_2) \in R(3)$. Then by Theorem 3.10 the constrained Cournot equilibrium strategies are $(q_i^*, e_i^*) = (\frac{a-c}{3}, \frac{3A_i}{a-c},)$. Then the social welfare is

$$\begin{aligned} W &= (a-c) \cdot \frac{2(a-c)}{3} - \frac{1}{2} \cdot \left(\frac{2(a-c)}{3}\right)^2 - D(A_1 + A_2) \\ &= \frac{4}{9} \cdot (a-c)^2 - D(A_1 + A_2). \end{aligned}$$

This completes the proof. □

Proof of Proposition 3.15. By Theorem 3.13 the social welfare function in $R(3)$ is

$$W(A_1, A_2) = \frac{4}{9} \cdot (a-c)^2 - D(A_1 + A_2).$$

By Assumption 3.6, D is strictly increasing with $A_1 + A_2$ and $D(0) = 0$. Thus, the maximum social welfare in $R(3)$ is $W = \frac{4}{9}(a-c)^2$ under allocation $(A_1, A_2) = (0, 0)$. □

Proof of Proposition 3.16. By Theorem 3.13 the social welfare function in $R(2a)$ is

$$W(A_1, A_2) = \frac{1}{2}(a-c) \cdot (a - c + \frac{A_1}{\alpha_1}) - \frac{1}{8} \cdot (a - c + \frac{A_1}{\alpha_1})^2 - D(A_1 + A_2).$$

In this function, only $D(A_1 + A_2)$ is related to variable A_2. Thus, according to Assumption 3.6, the maximum social welfare in $R(2a)$ can only occur when $A_2 = 0$. Then also $A_1 \geq \frac{\alpha_1 \cdot (a-c)}{3}$ by Remark 3.11. □

Proof of Theorem 3.18. By Lemma 3.26 we know that $(0, 0) > W(E_1^c, E_2^c)$ precisely when $d > d_0$. We argue that at least one of the two allowances $(0, 0)$ and (E_1^c, E_2^c) induces a strictly higher social welfare than any other carbon allowance.

Let (A_1, A_2) be an allocation of allowances. By Lemma 3.33 we know that $(A_1, A_2) \notin R(2)$. If $(A_1, A_2) \in R(3)$. Then $(A_1, A_2) = (0, 0)$ by Proposition 3.15. If $(A_1, A_2) \in R(1)$. Then

$$(A_1, A_2) = \begin{cases} (E_1^c, E_2^c) & \text{if } d \leq \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \\ (A^1, E_2^c) & \text{if } d > \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}. \end{cases}$$

by Lemma 3.23. However, suppose that $(A^1, E_2^c) \in R(1)$ and that $(A_1, A_2) = (A^1, E_2^c)$. Then, since $a - c \geq T$, by Lemma 3.29 and Lemma 3.30 it holds that

$$\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \geq \frac{B - \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}}{3\alpha_1^2} \geq d.$$

Then $(A^1, E_2^c) \notin R(1)$ by Lemma 3.25. This contradicts the assumption that $(A^1, E_2^c) \in R(1)$. This completes the proof. $\square$

Proof of Theorem 3.19. Suppose that $\alpha_1 \leq 2 \cdot \alpha_2$, and that $a - c < T$. Let (A_1, A_2) be an allocation of allowances. By Lemma 3.33 we know that $(A_1, A_2) \notin R(2)$. If $(A_1, A_2) \in R(3)$. Then $(A_1, A_2) = (0, 0)$ by Proposition 3.15. If $(A_1, A_2) \in R(1)$. Then

$$(A_1, A_2) = \begin{cases} (E_1^c, E_2^c) & \text{if } d \leq \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \\ (A^1, E_2^c) & \text{if } d > \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}. \end{cases}$$

by Lemma 3.23. So, we only need to compare $W(0, 0)$ with $W(E_1^c, E_2^c)$ and $W(A^1, E_2^c)$, respectively.

If $d \leq \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}$. We only need to compare (E_1^c, E_2^c) and $(0, 0)$. Note that

$$d \leq \frac{a - c - \alpha_1 - \alpha_2}{3\alpha_1} < d_0$$

by Lemma 3.31. So, by Lemma 3.26 we know that $W(0, 0) < W(E_1^c, E_2^c)$. Hence, $(A_1, A_2) = (E_1^c, E_2^c)$ in this case.

If $\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} < d \leq d_1$. Since

$$\frac{a - c - \alpha_1 - \alpha_2}{3\alpha_1} < d_1$$

by Lemma 3.30, the interval $I = \left[\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}, d_1\right]$ is not empty. By Lemma 3.32 it holds that

$$\frac{2 \cdot (a - c) + \alpha_1 - 2\alpha_2}{6\alpha_1} \geq d_1.$$

So, by Lemma 3.25 we know that $(A^1, E_2^c) \in R(1)$ for every $d \in I$. Further, since $d \leq d_1$, by Lemma 3.29 we know that $W(A^1, E_2^c) \geq W(0, 0)$. Hence, $(A_1, A_2) = (A^1, E_2^c)$ in this case.

If $d \geq d_1$. Then $W(A^1, E_2^c) < W(0,0)$ by Lemma 3.29. It follows that $(A_1, A_2) = (0,0)$ in this case. □

3.7.1 Partial results I: Identification of candidates

Lemma 3.22. *Let (A_1, A_2) be a government-optimal allocation of allowances. If $(A_1, A_2) \in R(2a)$, then*

$$(A_1, A_2) = \begin{cases} (A(2a), 0) & \text{if } \frac{\alpha_1}{3}(a-c) < A(2a) < E_1^c \\ (E_1^c, 0) & \text{if } E_1^c \leq A(2a). \end{cases}$$

A similar statement holds for the government-optimal allowance within $R(2b)$.

Proof. By Theorem 3.13 the social welfare function in $R(2a)$ is

$$W(A_1, A_2) = \frac{1}{2}\cdot(a-c)\cdot\left(a - c + \frac{A_1}{\alpha_1}\right) - \frac{1}{8}\cdot\left(a - c + \frac{A_1}{\alpha_1}\right)^2 - d\cdot(A_1 + A_2).$$

Since A_2 is only present in the term related to carbon emission, clearly $A_2 = 0$ is optimal. The resulting social welfare function is a concave quadratic function of only A_1. The first order condition is

$$\frac{\partial W}{\partial A_1} = \frac{a-c}{2}\cdot\frac{1}{\alpha_1} - \frac{2}{8}\left(a - c + \frac{A_1}{\alpha_1}\right)\cdot\frac{1}{\alpha_1} - d = 0,$$

with solution $A(2a) = \alpha_1 \cdot (a - c - 4\alpha_1 d)$. Thus, if $\frac{\alpha_1}{3}(a-c) \leq A(2a) \leq E_1^c$, the maximum social welfare is achieved at interior point $(\alpha_1(a - c - 4\alpha_1 d), 0)$. If $A(2a) < \frac{\alpha_1}{3}(a-c)$, then the maximum location would be $\left(\frac{\alpha_1}{3}(a-c), 0\right)$. However, this allocation of allowances is not government-optimal due to Proposition 3.15. If $A(2a) \geq E_1^c$, then $(E_1^c, 0)$ maximizes $W(A_1, A_2)$ over $R(2a)$. Similar considerations yield the result for $R(2b)$. □

We write $A^1 = \alpha_1 \cdot \left(\frac{2(a-c)-2\alpha_2+\alpha_1}{3} - \alpha_1 \cdot d\right)$.

Lemma 3.23. *Let (A_1, A_2) be a government-optimal allocation of allowances. If $(A_1, A_2) \in R(1)$, then*

$$(A_1, A_2) = \begin{cases} (E_1^c, E_2^c) & \text{if } d \leq \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \\ (A^1, E_2^c) & \text{if } d > \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}. \end{cases}$$

If $(A_1, A_2) \in R(1)$, and $\alpha_1 = \alpha_2 = \alpha$, then $A_1 + A_2 = \alpha \cdot (a - c) - d \cdot \alpha^2$.

Proof. By Theorem 3.13 the social welfare function in $R(1)$ is

$$W(A_1, A_2) = (a - c) \cdot \left(\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2}\right) - \frac{1}{2} \cdot \left(\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2}\right)^2 - d \cdot (A_1 + A_2).$$

For fixed $S > 0$, the social welfare function along the line $\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2} = S$ can be written as

$$W = (a - c) \cdot S - \frac{1}{2} \cdot S^2 - d \cdot (A_1 + A_2).$$

Thus, maximizing W along this line is equivalent to minimizing the objective function $A_1 + A_2$ along this line. Then the maximum location of the optimization problem is the point on the line segment where A_1 is minimized, unless $\alpha_1 = \alpha_2$, in which case every point on the line segment is an optimal point. Further, the points on the line segment $\frac{A_i}{\alpha_i} + 2\frac{A_j}{\alpha_j} = a - c$ are never optimal according to Lemma 3.22.

Thus, we only need to consider the case $A_2 = E_2^c$ if $\alpha_1 > \alpha_2$, and the case where $\alpha_1 = \alpha_2$. We further analyze this optimization problem in these two cases separately.

I. If $\alpha_1 > \alpha_2$. The maximum social welfare can only occur when $A_2 = E_2^c$. Then $A_1 = \alpha_1 \cdot \left(S - \frac{E_2^c}{\alpha_2}\right)$. Then

$$W = (a - c) \cdot S - \frac{1}{2} \cdot S^2 - d \cdot \alpha_1 \cdot \left(S - \frac{A_2}{\alpha_2}\right) - d \cdot E_2^c.$$

Maximizing over S yields $S = a-c-d\cdot\alpha_1$. Thus, the optimal allocation is

$$(A_1, A_2) = \left(\alpha_1 \cdot \left(S - \frac{E_2^c}{\alpha_2}\right), E_2^c\right) \quad \text{with } S = a - c - d \cdot \alpha_1.$$

II. If $\alpha_1 = \alpha_2 = \alpha$. Then the social welfare function is

$$W = \frac{a-c}{\alpha} \cdot (A_1 + A_2) - \frac{1}{2\alpha^2} \cdot (A_1 + A_2)^2 - d \cdot (A_1 + A_2).$$

The maximum social welfare is then achieved along $A_1 + A_2 = \alpha \cdot (a - c) - d \cdot \alpha^2$. □

3.7.2 Partial results II: Brackets for damage coefficient

Lemma 3.24. *It holds that* $(A(2a), 0) \in R(2a)$ *precisely when*

$$\frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1 - \alpha_2}{12\alpha_1} < d \leq \frac{a-c}{6\cdot\alpha_1}.$$

Proof. Suppose that $(A(2a), 0) \in R(2a)$. Then

$$A(2a) < \alpha_1 \cdot \frac{a - c + 2\alpha_1 - \alpha_2}{3} \quad \text{and} \quad A(2a) \geq \frac{\alpha_1}{3}(a-c),$$

where $A(2a) = \alpha_1 \cdot (a - c - 4\alpha_1 d)$. So, this is equivalent to

$$a - c \leq 3 \cdot (a - c - 4\alpha_1 d) < a - c + 2\alpha_1 - \alpha_2.$$

Which is equivalent to

$$\frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1 - \alpha_2}{12\alpha_1} < d \leq \frac{a-c}{6\cdot\alpha_1}.$$

If $2(a-c) - (2\alpha_1 - \alpha_2) > 0$. Otherwise, $d \geq 0$. This completes the proof. □

Lemma 3.25. *If $E_2^c \geq \alpha_2 \cdot \frac{a-c}{3}$. It holds that $(A^1, E_2^c) \in R(1)$ precisely when*

$$\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} \leq d \leq \frac{2\cdot(a-c)+\alpha_1-2\alpha_2}{6\alpha_1}.$$

Proof. If $E_2^c \geq \alpha_2 \cdot \frac{a-c}{3}$. By the definition of $R(1)$, we have that $(A^1, E_2^c) \in R(1)$ precisely when

$$2\alpha_2 A^1 + \alpha_1 E_2^c \geq \alpha_1 \cdot \alpha_2 \cdot (a-c) \quad \text{and} \quad A^1 \leq E_1^c.$$

So, since $A^1 = \alpha_1 \cdot \left(\frac{2(a-c)-2\alpha_2+\alpha_1}{3} - \alpha_1 \cdot d\right)$ and $E_2^c = \alpha_2 \cdot \frac{a-c+2\alpha_2-\alpha_1}{3}$, the first inequality is equivalent to

$$2 \cdot \left(\frac{2(a-c)-2\alpha_2+\alpha_1}{3} - \alpha_1 \cdot d\right) + \frac{a-c+2\alpha_2-\alpha_1}{3} \geq a-c.$$

Rewriting yields $6\alpha_1 \cdot d \leq 2\cdot(a-c)+\alpha_1-2\alpha_2$.

If $E_2^c \leq \frac{a-c}{3}$. We check when $\alpha_2 A^1 + 2\alpha_1 E_2^c \geq \alpha_1 \cdot \alpha_2 \cdot (a-c)$. This yields

$$\frac{2(a-c)-2\alpha_2+\alpha_1}{3} - \alpha_1 \cdot d + 2 \cdot \left(\frac{a-c+2\alpha_2-\alpha_1}{3}\right) \geq a-c.$$

So, $3\alpha_1 \cdot d \leq a-c+2\alpha_2-\alpha_1$.

The second inequality $A^1 \leq E_1^c$ is equivalent to

$$2(a-c)-2\alpha_2+\alpha_1-3\alpha_1 \cdot d \leq a-c+2\alpha_1-\alpha_2.$$

This can be rewritten to

$$d \geq \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}.$$

However, both first inequalities are an immediate consequence of the second one. This completes the proof. □

3.7.3 Partial results III: Comparisons of candidates

We present the results per region. First, we compare the social welfare under (E_1^c, E_2^c) and $(0,0)$.

Define

$$d_0 = \frac{2(a-c)\cdot(\alpha_1+\alpha_2) - (\alpha_1+\alpha_2)^2}{18\cdot(E_1^c+E_2^c)}.$$

Lemma 3.26. *It holds that $W(0,0) > W(E_1^c, E_2^c)$ precisely when $d > d_0$.*

Proof. We need to compare the social welfare of the allocations (E_1^c, E_2^c) and $(0,0)$. Using Theorem 3.13 we can compute that

$$W(0,0) = \frac{4}{9}(a-c)^2$$

and

$$W(E_1^c, E_2^c) = (a-c)\cdot\left(\frac{E_1^c}{\alpha_1}+\frac{E_2^c}{\alpha_2}\right) - \frac{1}{2}\cdot\left(\frac{E_1^c}{\alpha_1}+\frac{E_2^c}{\alpha_2}\right)^2 - d\cdot(E_1^c+E_2^c)$$

with $E_i^c = \alpha_i \cdot \frac{a-c+2\alpha_i-\alpha_j}{3}$. So, $W(0,0) \geq W(E_1^c, E_2^c)$ precisely when

$$\begin{aligned} 8\cdot(a-c)^2 &\geq 6(a-c)\cdot(2(a-c)+\alpha_1+\alpha_2) - (2(a-c)+\alpha_1+\alpha_2)^2 \\ &- 18d\cdot(E_1^c+E_2^c)\,. \end{aligned}$$

Expanding, and canceling all terms $(a-c)^2$ yields

$$4(a-c)\cdot(\alpha_1+\alpha_2) + (\alpha_1+\alpha_2)^2 \geq 6(a-c)\cdot(\alpha_1+\alpha_2) - 18d\cdot(E_1^c+E_2^c).$$

This is equivalent to

$$d \geq \frac{2(a-c)\cdot(\alpha_1+\alpha_2) - (\alpha_1+\alpha_2)^2}{18\cdot(E_1^c+E_2^c)} = d_0.$$

This completes the proof. □

Recall that $B = (2\alpha_1+\alpha_2)\cdot(a-c)+(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)$. Then we show when allocation (A^1, E_2^c) is superior to $(0,0)$ by Lemmas 3.27-3.29.

Lemma 3.27. *It holds that* $B > 0$.

Proof. By the Assumption we know that $\alpha_1 + \alpha_2 < a - c$. It follows that

$$\begin{aligned} B &= (2\alpha_1 + \alpha_2)\cdot(a-c) + (\alpha_1 - 2\alpha_2)\cdot(\alpha_1 - \alpha_2) \\ &> (2\alpha_1 + \alpha_2)\cdot(\alpha_1 + \alpha_2) + (\alpha_1 - 2\alpha_2)\cdot(\alpha_1 - \alpha_2) \\ &= 2\alpha_1^2 + 3\alpha_1\alpha_2 + \alpha_2^2 + \alpha_1^2 - 3\alpha_1\alpha_2 + 2\alpha_2^2 \\ &= 3\alpha_1^2 + 3\alpha_2^2 > 0. \end{aligned}$$

This completes the proof. □

Lemma 3.28. *It holds that*

$$\frac{2\cdot(a-c) - 2\alpha_2 + \alpha_1}{6\alpha_1} \leq \frac{B + \sqrt{B^2 - \alpha_1^2\cdot(a-c)^2}}{3\alpha_1^2}.$$

Proof. We have that

$$\frac{2\cdot(a-c) - 2\alpha_2 + \alpha_1}{6\alpha_1} \leq \frac{B + \sqrt{B^2 - \alpha_1^2\cdot(a-c)^2}}{3\alpha_1^2}$$

is equivalent to

$$\alpha_1\cdot[2\cdot(a-c) - 2\alpha_2 + \alpha_1] \leq 2B + 2\cdot\sqrt{B^2 - \alpha_1^2\cdot(a-c)^2}.$$

We compute that

$$\begin{aligned} \alpha_1\cdot[2\cdot(a-c) - 2\alpha_2 + \alpha_1] &= 2\alpha_1\cdot(a-c) + \alpha_1\cdot(\alpha_1 - 2\alpha_2) \\ &= B - \alpha_2\cdot(a-c) + \alpha_2\cdot(\alpha_1 - \alpha_2) \\ &= B - \alpha_2\cdot(a - c - \alpha_1 + \alpha_2). \end{aligned}$$

So, if $a - c - \alpha_1 + \alpha_2 \geq 0$. Then, since $B \geq 0$,

$$\alpha_1 \cdot [2 \cdot (a-c) - 2\alpha_2 + \alpha_1] \leq B \leq 2B + 2 \cdot \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}.$$

If $a - c - \alpha_1 + \alpha_2 < 0$, then

$$\begin{aligned} \alpha_1 \cdot [2 \cdot (a-c) - 2\alpha_2 + \alpha_1] &= B - \alpha_2 \cdot (a - c - \alpha_1 + \alpha_2) \\ &= B + \alpha_2 \cdot (\alpha_1 - \alpha_2 - (a-c)) \\ &\leq B + \alpha_1 \cdot (\alpha_1 - \alpha_2 + (a-c)) \\ &= B + \alpha_1 \cdot (a-c) + \alpha_1 \cdot (\alpha_1 - \alpha_2) \\ &\leq B + B \\ &\leq 2B + 2 \cdot \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}. \end{aligned}$$

This completes the proof. □

Lemma 3.29. *Suppose that* $(A^1, E_2^c) \in R(1)$. *Then* $W(A^1, E_2^c) < W(0,0)$ *precisely when*

$$d > \frac{B - \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}}{3\alpha_1^2},$$

where $B = (2\alpha_1 + \alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (\alpha_1 - \alpha_2)$.

Proof By Theorem 3.13, we know that

$$W(0,0) = \frac{4}{9} \cdot (a-c)^2.$$

Further, by Theorem 3.13, and using $S = a - c - \alpha_1 \cdot d$,

$$\begin{aligned} W(A^1, E_2^c) &= (a-c) \cdot S - \frac{1}{2} \cdot S^2 - d \cdot (A^1 + E_2^c) \\ &= \frac{1}{2} \cdot (a-c)^2 - \frac{1}{2} \cdot \alpha_1^2 \cdot d^2 - d \cdot (A^1 + E_2^c). \end{aligned}$$

Here, $A^1 = \alpha_1 \cdot \left(\frac{2(a-c) - 2\alpha_2 + \alpha_1}{3} - \alpha_1 \cdot d \right)$. So, $W(A^1, E_2^c) < W(0,0)$

precisely when

$$9 \cdot \alpha_1^2 \cdot d^2 + 18 \cdot d \cdot (A^1 + E_2^c) - (a-c)^2 > 0.$$

which is equivalent to

$$9 \cdot \alpha_1^2 \cdot d^2 - 6 \cdot d \cdot \Big[\alpha_1 \cdot (2(a-c) - 2\alpha_2 + \alpha_1) + \alpha_2 \cdot (a - c + 2\alpha_2 - \alpha_1)\Big] + (a-c)^2 < 0.$$

Which is equivalent to

$$9 \cdot \alpha_1^2 \cdot d^2 - 6 \cdot d \cdot \Big[(2\alpha_1 + \alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (\alpha_1 - \alpha_2)\Big] + (a-c)^2 < 0.$$

When we write $B = (2\alpha_1 + \alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (\alpha_1 - \alpha_2)$, this simplifies to

$$9 \cdot \alpha_1^2 \cdot d^2 - 6 \cdot d \cdot B + (a-c)^2 < 0.$$

This is equivalent to

$$\frac{B - \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}}{3\alpha_1^2} < d < \frac{B + \sqrt{B^2 - \alpha_1^2 \cdot (a-c)^2}}{3\alpha_1^2}.$$

However, since $(A^1, E_2^c) \in R(1)$, by Lemma 3.25, we have that

$$d \leq \frac{a - c - 2\alpha_2 + \alpha_1}{6\alpha_1}.$$

Then by Lemma 3.28 and the assumption that $B \geq 0$, the result follows. □

In what follows, Lemma 3.30 to Lemma 3.32 compare the boundaries and brackets for each candidate.

Let T be equal to $\frac{\alpha_2 \cdot (5\alpha_1 - \alpha_2) + \sqrt{\alpha_2^2 \cdot (5\alpha_1 - \alpha_2)^2 + 2 \cdot (\alpha_1 + \alpha_2)^2 \cdot [3\alpha_1^2 - 5\alpha_1\alpha_2 + 4\alpha_2^2]}}{2 \cdot (\alpha_1 + \alpha_2)}$.

Lemma 3.30. *It holds that*

$$\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} < \frac{B-\sqrt{B^2-\alpha_1^2\cdot(a-c)^2}}{3\alpha_1^2}$$

precisely when $a-c<T$.

Proof. The inequality is equivalent to

$$\alpha_1\cdot[a-c-\alpha_1-\alpha_2] < B-\sqrt{B^2-\alpha_1^2\cdot(a-c)^2}.$$

This is equivalent to

$$\sqrt{B^2-\alpha_1^2\cdot(a-c)^2} < B-\alpha_1\cdot[a-c-\alpha_1-\alpha_2]\,,$$

while $B=(2\alpha_1+\alpha_2)\cdot(a-c)+(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)$. Note that

$$B-\alpha_1\cdot[a-c-\alpha_1-\alpha_2]=(\alpha_1+\alpha_2)\cdot(a-c)+(2\alpha_1^2-2\alpha_1\alpha_2+2\alpha_2^2)>0.$$

So, squaring both sides of the inequality, omitting a term B^2 on both sides, and multiplying by -1 yields

$$\alpha_1^2\cdot(a-c)^2 > 2B\cdot\alpha_1\cdot[a-c-\alpha_1-\alpha_2]-\alpha_1^2\cdot[a-c-\alpha_1-\alpha_2]^2$$

We substitute $Z=a-c$, and omit a common factor α_1. This yields

$$\alpha_1\cdot Z^2 > 2B\cdot[Z-\alpha_1-\alpha_2]-\alpha_1\cdot[Z-\alpha_1-\alpha_2]^2$$

where $B=(2\alpha_1+\alpha_2)\cdot Z+(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)$.

We obtain

$$\begin{array}{rl} & [\alpha_1-2\cdot(2\alpha_1+\alpha_2)]\cdot Z^2 \\ > & 2\cdot(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)\cdot Z-2\cdot(2\alpha_1+\alpha_2)\cdot(\alpha_1+\alpha_2)\cdot Z \\ - & 2\cdot(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)\cdot(\alpha_1+\alpha_2)-\alpha_1\cdot[Z-\alpha_1-\alpha_2]^2 \end{array}$$

Then we can get that $-(2\alpha_2+2\alpha_1)\cdot Z^2$ is gather than the following

equation:

$$\begin{aligned} & - \quad 2(2\alpha_1+\alpha_2)(\alpha_1+\alpha_2) - 2(\alpha_1-2\alpha_2)(\alpha_1-\alpha_2)(\alpha_1+\alpha_2)\cdot Z \\ & + \quad 2\alpha_1(\alpha_1+\alpha_2)\cdot Z + 2(\alpha_1-2\alpha_2)(\alpha_1-\alpha_2)\cdot Z - \alpha_1(\alpha_1+\alpha_2)^2. \end{aligned}$$

This yields

$$\begin{aligned} & - \quad (2\alpha_2+2\alpha_1)\cdot Z^2 \\ & > \quad 2\cdot(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)\cdot Z - 2\cdot(\alpha_1+\alpha_2)\cdot(\alpha_1+\alpha_2)\cdot Z \\ & - \quad 2\cdot(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)\cdot(\alpha_1+\alpha_2) - \alpha_1\cdot(\alpha_1+\alpha_2)^2. \end{aligned}$$

Collecting terms, and multiplying by -1 yields

$$(2\alpha_2+2\alpha_1)\cdot Z^2 < 2\cdot\alpha_2\cdot(5\alpha_1-\alpha_2)\cdot Z + (\alpha_1+\alpha_2)\cdot[3\alpha_1^2-5\alpha_1\alpha_2+4\alpha_2^2].$$

Thus, the inequality can be written as

$$A\cdot Z^2 - D\cdot Z - C < 0,$$

where $A = 2\cdot(\alpha_1+\alpha_2)$, $D = 2\cdot\alpha_2\cdot(5\alpha_1-\alpha_2)$, and $C = (\alpha_1+\alpha_2)\cdot[3\alpha_1^2-5\alpha_1\alpha_2+4\alpha_2^2]$. We compute that

$$D^2 + 4\cdot A\cdot C = 4\cdot\alpha_2^2\cdot(5\alpha_1-\alpha_2)^2 + 8\cdot(\alpha_1+\alpha_2)^2\cdot[3\alpha_1^2-5\alpha_1\alpha_2+4\alpha_2^2].$$

Since this expression is strictly larger than zero, and also (even) strictly larger than D^2, it follows that the inequality is valid precisely when

$$Z < \frac{D+\sqrt{D^2+4AC}}{2A},$$

which is $\frac{\alpha_2\cdot(5\alpha_1-\alpha_2)+\sqrt{\alpha_2^2\cdot(5\alpha_1-\alpha_2)^2+2\cdot(\alpha_1+\alpha_2)^2\cdot[3\alpha_1^2-5\alpha_1\alpha_2+4\alpha_2^2]}}{2\cdot(\alpha_1+\alpha_2)}$.

This completes the proof. □

Lemma 3.31. *It holds that*

$$\frac{a-c-\alpha_1-\alpha_2}{3\alpha_1} < d_0$$

precisely when $a-c<T$.

Proof. Write $d^* = \frac{a-c-\alpha_1-\alpha_2}{3\alpha_1}$. Note that, at d^*, it holds that $A^1 = E_1^c$. In particular, we have that $W^*(E_1^c, E_2^c) = W^*(A^1, E_2^c)$ holds at d^*, where W^* indicates the welfare function for the choice $d = d^*$.

Consider the inequality $d^* < d_0$. According to Lemma 3.26, this is equivalent to $W(0,0) < W^*(E_1^c, E_2^c) = W^*(A^1, E_2^*)$. According to Lemma 3.29, this is equivalent to

$$d^* < \frac{B-\sqrt{B^2-\alpha_1\cdot(a-c)^2}}{3\alpha_1^2}.$$

This is equivalent to $a-c<T$ by Lemma 3.30. □

Lemma 3.32. *It holds that*

$$\frac{2\cdot(a-c)+\alpha_1-2\alpha_2}{6\alpha_1} > \frac{B-\sqrt{B^2-\alpha_1^2\cdot(a-c)^2}}{3\alpha_1^2}.$$

Proof. The inequality is equivalent to

$$\alpha_1\cdot[2\cdot(a-c)+\alpha_1-2\alpha_2] > 2\cdot\left[B-\sqrt{B^2-\alpha_1^2\cdot(a-c)^2}\right].$$

This is equivalent to

$$2\cdot\sqrt{B^2-\alpha_1^2\cdot(a-c)^2} > 2\cdot B-\alpha_1\cdot[2\cdot(a-c)+\alpha_1-2\alpha_2],$$

while $B = (2\alpha_1+\alpha_2)\cdot(a-c)+(\alpha_1-2\alpha_2)\cdot(\alpha_1-\alpha_2)$. Note that $2\cdot B-\alpha_1\cdot[2\cdot(a-c)+\alpha_1-2\alpha_2] = (2\alpha_1+2\alpha_2)\cdot(a-c)+(\alpha_1-2\alpha_2)^2 > 0$.

So, indeed the right hand side of the inequality is strictly positive. So, squaring both sides, and omitting a term $4B^2$ and a factor α_1, yields the inequality $-4 \cdot \alpha_1 \cdot (a-c)^2 \geq -4B \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2] + \alpha_1 \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2]^2$.

Which is equivalent to

$$4B \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2] - \alpha_1 \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2]^2 > 4 \cdot \alpha_1 \cdot (a-c)^2$$

Which is equivalent to

$$[2(a-c) + \alpha_1 - 2\alpha_2] \cdot (4B - \alpha_1 \cdot [2(a-c) + \alpha_1 - 2\alpha_2]) > 4\alpha_1 \cdot (a-c)^2.$$

We have that

$$\begin{aligned} & 4B - \alpha_1 \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2] \\ = \; & 4 \cdot [(2\alpha_1 + \alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (\alpha_1 - \alpha_2)] \\ & -\alpha_1 \cdot [2 \cdot (a-c) + \alpha_1 - 2\alpha_2] \\ = \; & (8\alpha_1 + 4\alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (4\alpha_1 - 4\alpha_2) \\ & -2\alpha_1 \cdot (a-c) - \alpha_1 \cdot (\alpha_1 - 2\alpha_2) \\ = \; & (6\alpha_1 + 4\alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (3\alpha_1 - 4\alpha_2). \end{aligned}$$

So, expanding the left hand side

$$[2(a-c) + \alpha_1 - 2\alpha_2] \cdot [(6\alpha_1 + 4\alpha_2) \cdot (a-c) + (\alpha_1 - 2\alpha_2) \cdot (3\alpha_1 - 4\alpha_2)]$$

yields

$$\begin{aligned} & (12\alpha_1 + 8\alpha_2)(a-c)^2 \\ + \; & (\alpha_1 - 2\alpha_2) \left[(12\alpha_1 - 4\alpha_2)(a-c) + (3\alpha_1 - 4\alpha_2)(\alpha_1 - 2\alpha_2) \right]. \end{aligned}$$

Subtracting $4\alpha_1 \cdot (a-c)^2$ yields

$$\begin{aligned} & (8\alpha_1 + 8\alpha_2)(a-c)^2 \\ + \; & (\alpha_1 - 2\alpha_2) \left[(12\alpha_1 - 4\alpha_2)(a-c) + (3\alpha_1 - 4\alpha_2)(\alpha_1 - 2\alpha_2) \right]. \end{aligned}$$

Thus, the inequality is quadratic in $z = a - c$. We can write

$$A \cdot z^2 - B \cdot z - C > 0,$$

where $z = a - c$, $A = 8 \cdot (\alpha_1 + \alpha_2)$, $B = (2\alpha_2 - \alpha_1) \cdot (12\alpha_1 - 4\alpha_2)$, and $C = (4\alpha_2 - 3\alpha_1) \cdot (2\alpha_2 - \alpha_1)^2$. We argue that the inequality is valid. Write

$$[A \cdot z - B] \cdot z - C > 0.$$

Note that $a - c \geq \alpha_1 + \alpha_2$, so that

$$A \cdot z - B \geq A \cdot (\alpha_1 + \alpha_2) - B = 20 \cdot \alpha_1^2 - 12 \cdot \alpha_1 \cdot \alpha_2 + 16 \cdot \alpha_2^2.$$

Further, $4\alpha_2 - 3\alpha_1 \leq 2\alpha_1 + 2\alpha_2$, so that

$$C \leq 2 \cdot (\alpha_1 + \alpha_2) \cdot (2\alpha_2 - \alpha_1)^2.$$

So, since indeed $A \cdot z - B > 0$, it follows that

$$\begin{array}{ll} & A \cdot z^2 - B \cdot z - C \\ = & [A \cdot z - B] \cdot z - C \\ \geq & \left[20 \cdot \alpha_1^2 - 12 \cdot \alpha_1 \cdot \alpha_2 + 16 \cdot \alpha_2^2\right] \cdot (a - c) - 2 \cdot (\alpha_1 + \alpha_2) \cdot (2\alpha_2 - \alpha_1)^2. \end{array}$$

So, again using the fact that $a - c \geq \alpha_1 + \alpha_2$, it suffices to show that

$$\left[20 \cdot \alpha_1^2 - 12 \cdot \alpha_1 \cdot \alpha_2 + 16 \cdot \alpha_2^2\right] - 2 \cdot (2\alpha_2 - \alpha_1)^2 > 0.$$

A straightforward computation shows that this inequality equals

$$18 \cdot \alpha_1^2 - 4 \cdot \alpha_1 \cdot \alpha_2 + 8 \cdot \alpha_2^2 > 0.$$

The left hand side is a convex quadratic form with a negative discriminant. Hence, this expression is indeed strictly larger than zero. This completes the proof. □

Write $d^* = \frac{a-c}{6 \cdot \alpha_1} - \frac{2\alpha_1 - \alpha_2}{12\alpha_1}$. Then we show that the government-optimal policy is not in $R(2)$.

Lemma 3.33. *Let (A_1, A_2) be a government-optimal allocation of allowances. Then $(A_1, A_2) \notin R(2)$.*

Proof. Let (A_1, A_2) be a government-optimal allocation of allowances. Suppose that $(A_1, A_2) \in R(2a)$. Then by Lemma 3.22

$$(A_1, A_2) = \begin{cases} (A(2a), 0) & \text{if } \frac{\alpha_1}{3}(a-c) < A(2a) < E_1^c \\ (E_1^c, 0) & \text{if } E_1^c \leq A(2a). \end{cases}$$

If $(A_1, A_2) = (A(2a), 0)$. Then by Lemma 3.34 it holds that $W^*(E_1^c, 0) \geq W(A(2a), 0)$, where W^* corresponds to $d^* = \frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1-\alpha_2}{12\alpha_1}$. Then, by Lemma 3.24, it suffices to show that $W(0,0) > W^*(E_1^c, 0)$.

Take any $d \geq 0$. Since $\alpha_1 + \alpha_2 \leq a - c$ and $\alpha_1 \leq 2 \cdot \alpha_2$ we have that

$$2\alpha_1 - \alpha_2 \leq \alpha_1 + \alpha_2 < a - c \leq (\sqrt{5} - 1) \cdot (a - c).$$

Then from Lemma 3.35 it follows that $W(0,0) > W^*(E_1^c, 0)$. A similar proof holds for the region $R(2b)$. □

Lemma 3.34. *Suppose that $(A(2a), 0) \in R(2a)$. Then we have*

$$W^*(E_1^c, 0) \geq W(A(2a), 0),$$

where W^* corresponds to d^*.

Proof. Since $(A(2a), 0) \in R(2a)$, by Theorem 3.13 we have that

$$\begin{aligned} W(A(2a), 0) &= \frac{1}{2}(a-c) \cdot (2(a-c) - 4\alpha_1 d) \\ &- \frac{1}{8}(2(a-c) - 4\alpha_1 d)^2 - d \cdot A(2a) \\ &= \frac{1}{2}(a - c - 2\alpha_1 d) \cdot (a - c + 2\alpha_1 d) - d \cdot A(2a) \\ &= \frac{1}{2}(a-c)^2 - 2\alpha_1^2 d^2 - d \cdot A(2a) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}(a-c)^2 - 2\alpha_1^2 d^2 - d\cdot\alpha_1\cdot(a-c-4\alpha_1 d) \\ &= \frac{1}{2}(a-c)^2 + 2\alpha_1^2 d^2 - d\cdot\alpha_1\cdot(a-c). \end{aligned}$$

Moreover,

$$\frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1-\alpha_2}{12\alpha_1} < d \leq \frac{a-c}{6\cdot\alpha_1}$$

by Lemma 3.24. Then, since $d \leq \frac{a-c}{6\cdot\alpha_1}$, we have that

$$W'(d) = 4\alpha_1^2\cdot d - \alpha_1\cdot(a-c) < 0,$$

so that $W(d)$ is strictly decreasing on the interval

$$\frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1-\alpha_2}{12\alpha_1} < d \leq \frac{a-c}{6\cdot\alpha_1}.$$

It follows that

$$W(d) \leq W\left(d^*\right)$$

for $d^* = \frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1-\alpha_2}{12\alpha_1}$. However, for d^*,

$$\begin{aligned} A(2a) &= \alpha_1\cdot(a-c-4\alpha_1\cdot d^*) \\ &= \alpha_1\cdot\left(a-c-4\alpha_1\cdot\left(\frac{a-c}{6\cdot\alpha_1} - \frac{2\alpha_1-\alpha_2}{12\alpha_1}\right)\right) \\ &= \alpha_1\cdot\left(a-c-\frac{2}{3}\cdot(a-c) + \frac{1}{3}\cdot(2\alpha_1-\alpha_2)\right) \\ &= \alpha_1\cdot\left(\frac{1}{3}\cdot(a-c) + \frac{1}{3}\cdot(2\alpha_1-\alpha_2)\right) = E_1^c. \end{aligned}$$

It follows that, for any d in the domain,

$$W(A(2a),0) = W(d) \leq W(d^*) = W^*(E_1^c,0).$$

This completes the proof. □

Lemma 3.35. *It holds that* $W(0,0) > W^*(E_1^c, 0)$ *precisely when*

$$2\alpha_1 - \alpha_2 < (\sqrt{5} - 1) \cdot (a - c).$$

Proof. Note that $(E_1^c, 0) \in R(2a)$. So, by Theorem 3.13 we know that

$$\begin{aligned} W^*(E_1^c, 0) &= \frac{1}{2} \cdot (a-c) \cdot \frac{4(a-c) + 2\alpha_1 - \alpha_2}{3} \\ &\quad -\frac{1}{8} \cdot \left[\frac{4(a-c) + 2\alpha_1 - \alpha_2}{3}\right]^2 - \frac{\alpha_1}{3} \cdot d^* \cdot (a - c + 2\alpha_1 - \alpha_2) \\ &= \frac{1}{72} \cdot [12 \cdot (a-c) \cdot (4(a-c) + 2\alpha_1 - \alpha_2) \\ &\quad -(4(a-c) + 2\alpha_1 - \alpha_2)^2 - 24 \cdot \alpha_1 \cdot d^* \cdot (a - c + 2\alpha_1 - \alpha_2)] \\ &= \frac{1}{72} \cdot \left[32 \cdot (a-c)^2 - (2\alpha_1 - \alpha_2)^2 + 4(a-c) \cdot (2\alpha_1 - \alpha_2)\right. \\ &\quad \left. -2(2(a-c) - (2\alpha_1 - \alpha_2)) \cdot (a - c + 2\alpha_1 - \alpha_2)\right] \\ &= \frac{1}{72} \cdot \left[28(a-c)^2 + 2(a-c) \cdot (2\alpha_1 - \alpha_2) + (2\alpha_1 - \alpha_2)^2\right]. \end{aligned}$$

So, $W^*(E_1^c, 0) < W(0,0)$ precisely when

$$28(a-c)^2 + 2(a-c) \cdot (2\alpha_1 - \alpha_2) + (2\alpha_1 - \alpha_2)^2 < 32 \cdot (a-c)^2.$$

Rewriting yields

$$(2\alpha_1 - \alpha_2)^2 + 2(a-c) \cdot (2\alpha_1 - \alpha_2) - 4 \cdot (a-c)^2 < 0,$$

which is a quadratic expression in $2\alpha_1 - \alpha_2$. Solving yields that $0 \leq 2\alpha_1 - \alpha_2 < (\sqrt{5} - 1) \cdot (a - c)$. This completes the proof. □

4

A Dynamic Surge Pricing Model throughout Product Lifecycle

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Abstract

The shorter lifecycle and faster upgrading of the product make the potential demand change rapidly, thereby pricing based on market changes is critical to increasing profits. In response to changes in the potential demand, we present a dynamic surge pricing model that characterizes sales trajectories and gradient pricing. We introduce the Lotka-Volterra system to construct the sale-forecast system and prove its effectiveness in predicting the product lifecycle curve that reflects the change of the potential demand. In contrast to the linear demand function, we propose a gradient pricing mechanism based on marginal sales and total sales to describe the relationship between price and potential demand throughout the life cycle. Particularly, the dynamic surge pricing model degrades to Cournot model in the maturity phase of the market. We characterize the dynamic equilibrium and conduct the sensitivity analysis of parameters, showing that the dynamic surge pricing model outperforms Cournot model in terms of profit. A numerical example illustrates that the profit of the dynamic surge pricing model is nearly 21.82% higher than that of Cournot model.

4.1 Introduction

With the rise of e-commerce, products in the market, especially high-tech categories, take the characteristics of rapid change, platform dependence, strong substitution, and so on (Lin and Sibdari, 2009). Consequently, demand changes rapidly and creates a difficult but critical task of dynamic pricing based on real-time demand (Den Boer, 2015; AmalNick and Qorbanian, 2017). Dynamic pricing is a popular pricing strategy in which the price changes over time to adapt to the rapidly changing market demand (Cachon et al., 2017). For instance, Amazon's 2021 annual report showed that the product price was adjusted more than 2.5 million times one day, with net sales growing 22% higher than a year ago.

Most of the existing researches focus on the dynamic pricing of products by the linear demand function, which was originally proposed in Cournot model. Scholars have fully studied the dynamical properties of Cournot model, including stability, bifurcation, and even chaos (Agliari et al., 2016; Ren et al., 2020). Fanti et al. (2015) analyze the dynamics of a Cournot duopoly with general isoelastic demand (quasi-linear preferences) and quantity-setting firms. Considering product differentiation, Andaluz and Jarne (2015) analyze the Cournot Nash equilibria and find that the Nash equilibrium is more stable under quantity competition than under price competition, regardless of whether goods are complements or substitutes. C. Wang et al. (2024) add strategic adjustment costs to the dynamic n-person Cournot games with a linear demand function, and results show that the adjustment costs improve the stability of the original Cournot-Nash equilibrium. Beyond these findings, a series of studies on different factors and properties have been conducted, such as time delay (Ding et al., 2014), incomplete information (Ren et al., 2023), and R&D spillovers (Cao et al., 2022).

However, the Cournot demand function only reflects the relationship between price and output under fixed potential demand, which means that the market tends to mature without changing over time. In fact, as

shown in Figure 4.1, the potential demand is constantly changing due to product diffusion, word-of-mouth communication, advertising, etc (Routley et al., 2013). As a response to the change in potential demand, the product lifecycle curve describes the sales trajectory of a product, including the rapid growth phase, the slow growth phase, and the maturity phase (X. Li et al., 2021).

To increase profit, firms need to constantly change prices to match the potential demand for each period (Tao et al., 2021). For example, if the price is set low when the demand is growing fast, the profit will fall. If the price goes up when sales are slow, the goods won't sell, so firms will adopt a promotion strategy when the market becomes mature (Chernonog, 2020). For profit objectives, we should explore how to characterize the potential demand and how to make price decisions throughout the product lifecycle.

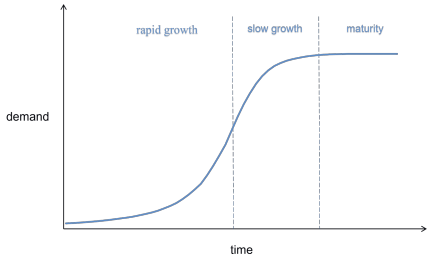


Figure 4.1: The product life cycle curve.

Motivated by the above issues, we develop a dynamic surge pricing model, which uses the Lotka-Volterra system for product lifecycle forecast and implements dynamic gradient pricing based on the system. We first explain the reasonability for introducing the Lotka-Volterra system to predict the product lifecycle curves of firms competing for the potential demand in a market. Originally proposed by Vito Volterra, the Lotka-Volterra system is a set of differential equations that simulate the population changes under the interaction between two species in the same ecosystem (G. Zhang et al., 2019). Due to

the similarity that both potential demand and ecological resources are limited, the Lotka-Volterra system can be applied to model the change of sales growth of two firms with different production capacity, competitiveness and capacity constraints (Yao et al., 2022). Therefore, we construct a sale-forecast system for product lifecycle forecast by the Lotka-Volterra system and show its effectiveness theoretically. The solution functions of the sale-forecast system are proved to be a set of rational product lifecycle functions, which is a new definition inspired by the production function.

Based on the sales forecast by the Lotka-Volterra system, we incorporate dynamic pricing into a gradient learning process to dynamically adjust prices throughout the life cycle. Prices vary with marginal sales and total sales, where total sales are the solution functions of the Lotka-Volterra system, and marginal sales correspond to the differential equations in the Lotka-Volterra system (i.e., the change in sales per unit time). Specifically, there are two basic rules for adjustment. One is that price increases when marginal sales are positive and decreases when marginal sales are negative. The other rule is that the price decreases as the total sales increase. We formulate the dynamic surge pricing model as a continuous-time dynamic system and derive sufficient conditions for the global asymptotic stability of equilibrium points of the system. Finally, we discuss the parameter conditions for the dynamic surge pricing equilibrium to be stable on Cournot equilibrium. Particularly, the price function degenerates into a linear inverse demand function in Cournot model during the market maturity period.

In the end, the value range and the influence on price and profit of each parameter of the dynamic surge pricing model are analyzed and a series of numerical experiments are presented to verify the theoretical results. Especially, a numerical example shows that the dynamic surge pricing model improves profit by nearly 21.82% compared with the Cournot model. The results indicate that dynamic pricing decisions considering the demand trend perform better than general pricing strategies, which complies with the data analysis by (Tong et al.,

2020).

The chapter is organized as follows. In Section 4.2, we construct the dynamic surge pricing model with sales forecast and surge price adjustment mechanism. The theoretical analysis of the dynamic surge pricing model is given in Section 4.3. Section 4.4 presents numerical experiments to analyze the properties of the dynamic surge pricing model. Section 4.5 presents the concluding remarks of the chapter. All proofs are collected in the Appendix.

4.2 The dynamic surge pricing model

The dynamic surge pricing model is constructed in two stages. First, a forecasting method of product lifecycle is proposed to respond to real-time demand. Then we develop a price gradient learning mechanism based on the sale-forecast system.

4.2.1 The sale-forecast system

Originally proposed by Vito Volterra, the Lotka-Volterra system is a set of differential equations that simulate the population dynamics under the interaction between two species in the same ecosystem. Given the similarity between potential demand and limited ecological resources, the Lotka-Volterra system can be applied to model the change in sales growth of two firms with different production capacities, competitiveness, and capacity constraints (Yao et al., 2022). Existing literature has used the Lotka-Volterra system as a standard model in this context (Ren et al., 2020). Through empirical analysis, H.-c. Teng and Y.-f. Huang (2013) demonstrated Lotka-Volterra system provides more accurate values than the traditional Grey forecasting model.

Based on the Lotka-Volterra system, we construct a sale-forecast system of firm i $(i = 1, 2)$ by the following differential equations,

$$\begin{cases} \frac{\mathrm{d}q_1}{\mathrm{d}t} = r_1 q_1 (1 - \frac{q_1}{k} - a_2 \frac{q_2}{k}), \\ \frac{\mathrm{d}q_2}{\mathrm{d}t} = r_2 q_2 (1 - \frac{q_2}{k} - a_1 \frac{q_1}{k}). \end{cases} \tag{4.1}$$

In system (4.1), $t \in [0, T]$ denotes the selling time, where T is the time of products out of stock. a_i measures the composition coefficient of enterprise i. The parameter r_i indicates the intrinsic growth rate for sales of enterprise i and k is the carrying capacity of each firm.

4.2.2 The surge price adjustment mechanism

According to the internal relationship between demand and price, we explore the way of dynamic pricing throughout the product lifecycle. We first review some preliminary knowledge about the classic Cournot model. Under the linear demand function in Cournot model, the dynamic price $(p_i^c(t))$ is expressed as

$$p_i^c(t) = a - \gamma(q_1^c(t) + q_2^c(t)),\ i = 1, 2.$$

However, the above decision-making method considers the impact of the current demand on price, but ignores the feedback control of the demand trend on price throughout the product lifecycle.

Considering the influence of dynamic demand on pricing, we propose a surge price adjustment mechanism based on total sales and marginal sales. Particularly, the marginal sales for an enterprise are the change in sales per unit of time, which is an immediate response to the potential demand change. According to the general assumption of bounded rationality (Elsadany, 2017), firms infer price decisions for the next period by the marginal sales $(\dot{q}_i(t))$ and total sales $(q_i(t))$, which can be derived from the system (4.1). The specific rules of price adjustment are explained below. First, the price is negatively correlated with total sales due to limited potential demand in the market. Second, based on

the trend of sales growth, the price is raised as marginal sales rise and is lowered otherwise. The gradient adjustment mechanism of price with the change of time for firm i $(i = 1, 2)$ can be obtained as,

$$\begin{cases} p_1(t) = p_1(0) + \lambda \cdot \dot{q}_1(t) - \mu \cdot (q_1(t) + q_2(t)), \\ p_2(t) = p_2(0) + \lambda \cdot \dot{q}_2(t) - \mu \cdot (q_1(t) + q_2(t)). \end{cases} \tag{4.2}$$

where λ and μ denote the coefficients of marginal sales and total sales, respectively. $p_i(0)$ is the initial price of enterprise i $(i = 1, 2)$ and $\dot{q}_i(t)$ is the forecast of marginal sales based on system (4.1).

We call system (4.2) the dynamic surge pricing model, which describes the dynamic competition between two firms in the markets. The dynamic surge pricing model provides a method to consider demand change in product pricing, and the properties of this model are analyzed in the next section.

4.3 Theoretical analysis of dynamic surge pricing model

As the dynamic surge pricing model is based on the sale-forecast system (4.1), the rationality of sale-forecast system (4.1) is proved in Subsection $4.3.1$.

4.3.1 The rationality of sales-forecast

The product sales can be regarded as the output under the input of time, which is similar to the concept of the production function in mathematical economics. Capturing the sales trajectories of products by the product lifecycle curve is inspired by the production function defined in Takayama and Akira, 1985, which states that it has the property of diminishing marginal utility. We forecast sales rationally by the following definition,

Definition 4.1. *A real-valued function $y = f(t)$ $(t \in [0, \infty))$ is called a product lifecycle curve if it satisfies the following conditions,*

(a) $\exists\, t_1 \in [0, \infty)$*, such that* $f(t)$ *monotonically increases when* $t < t_1$ *and is nonincreasing if* $t > t_1$*.*

(b) $\exists\, t_2 \in [0, \infty)$*, such that* $f(t)$ *is convex if* $t < t_2$ *and nonconvex if* $t > t_2$*.*

(c) The second derivative of the function is continuous.

Then the following theorem proves that the solution function of the differential system (4.1) is the product lifecycle curve under Definition 4.1.

Lemma 4.2. *Suppose that* $a_1 < 1$ *and* $a_2 < 1$*, then the corresponding solution functions* $q_i = q_i(t)$ *of system* (4.1) *are monotonically increasing if* $t \in (0, {q_i}^{-1}(k - a_j q_j))$ *and nonincreasing if* $t \in ({q_i}^{-1}(k - a_j q_j), \infty)$*, for* $i = 1, 2$*, respectively.*

According to Lemma 4.2, the assumption that $a_i < 1$ $(i = 1, 2)$ guarantees the first condition in Definition 4.1. This assumption is common in the literature that ensures the global asymptotic stability of the unique positive equilibrium point, which will be discussed in Subsection 4.3.2. In practice, a_i $(i = 1, 2)$ represents the competitiveness of firms. $a_i < 1$ $(i = 1, 2)$ means that the products of the two firms are not completely substituted, which means that for a firm, the sales growth of its rival has a limited restrain on its sales growth and will not exceed its own inhibiting effect.

Lemma 4.3. *Let* $a_1 < 1$ *and* $a_2 < 1$*, then there exist a unique* $t_2 \in [0, \infty)$ *such that the corresponding solution functions* $q_i = q_i(t)$ *of system* (4.1) *are convex when* $t \in (0, t_2)$ *and nonconvex* $t \in (t_2, \infty)$*, for* $i = 1, 2$*, respectively.*

Lemma 4.3 indicates that goods in the early stage sell faster and faster with the influence of advertisements and word-of-mouth. Next, we use Theorem 4.4 to show that system (4.4) can describe real-time demand when $a_i < 1$ $(i = 1, 2)$.

Theorem 4.4. *If $a_1 < 1$ and $a_2 < 1$, each solution function of system (4.1) is a product lifecycle curve which satisfies the condition (i),(ii) and (iii) in Definition 4.1.*

For the e-commerce market, a reasonable assumption of a company is that the production capacity is sufficient, that is, they can supply on demand. Therefore, the rational sale-forecast system ensures the balance of supply and demand in the market.

4.3.2 Equilibrium points of dynamic surge pricing model

This subsection explores the evolution of demand by answering the following questions: Does the dynamic surge pricing model have equilibrium points? Are these equilibrium points stable?

It is easy to know from the previous literature (Ren et al., 2020) that the equilibrium points of system (4.1) are $(0, 0)$, $(k, 0)$, $(0, k)$ and $(q_1^*, q_2^*) = (k(1-a_2)/(1-a_2a_1), k(1-a_1)/(1-a_2a_1))$. Then we give the equilibrium points of model (4.2).

Proposition 4.5. *The equilibrium points of dynamic surge pricing model (4.2) are $E_1 = (0, 0)$, $E_2 = (p_1(0) - \mu k, 0)$, $E_3 = (0, p_2(0) - \mu k)$ and $E_4 = (p_1^*, p_2^*)$, where*

$$(p_1^*, p_2^*) = (p_1(0) - \mu\frac{k(2 - a_2 - a_1)}{1 - a_2a_1}, p_2(0) - \mu\frac{k(2 - a_2 - a_1)}{1 - a_2a_1}).$$

Through system (4.1), we can calculate the stable points of the sales system and further derive the above stable points of model (4.2). E_1 represents that the product is not on the market yet. E_2 is stable if

$a_2 < 1, a_1 > 1$ and E_3 is stable if $a_2 > 1, a_1 < 1$, which indicates that one company occupies the whole market and the other is squeezed out of the market. The stable equilibrium point E_4 under the condition $a_2 < 1, a_1 < 1$ means that the two corporations compete and allocate market shares.

Now the sufficient conditions for global asymptotic stability of equilibrium point E_4 are discussed, which corresponds to the equilibrium point (q_1^*, q_2^*) in system (4.1).

Theorem 4.6. *For dynamic surge pricing model* (4.2)*, the unique positive equilibrium point* (p_1^*, p_2^*) *is globally asymptotically stable if* $a_1 < 1, a_2 < 1$*, where*

$$\begin{cases} \lim\limits_{t\to\infty} p_1(t) = p_1(0) - \mu\frac{k(2-a_2-a_1)}{1-a_2a_1} = p_1^*, \\ \lim\limits_{t\to\infty} p_2(t) = p_2(0) - \mu\frac{k(2-a_2-a_1)}{1-a_2a_1} = p_2^*. \end{cases}$$

The conditions for Theorem 4.6 and Theorem 4.4 are same, where $a_i < 1$ $(i = 1, 2)$ indicates that the competition between the two enterprises is not fierce. The global asymptotic stability in Theorem 4.6 indicates that the price is a relatively static state under the mutual restriction of a series of economic forces. Then we analyze the relationship between dynamic surge pricing equilibrium and Cournot equilibrium.

4.3.3 Equilibrium between dynamic surge pricing model and Cournot model

We continue to recall a conclusion about Cournot model, in which the equilibrium of firms 1 and 2 can be obtained as,

$$\begin{cases} q_1^{c*} = q_2^{c*} = \frac{a-c}{3\gamma}, \\ p_1^{c*} = p_2^{c*} = \frac{a+2c}{3}. \end{cases} \tag{4.3}$$

For the purposes of the study, we assume that the cost is zero, that is, $c = 0$. Then we can give Theorem 4.7 as follows, which describes the relationship of the equilibria between the dynamic surge pricing model and Cournot model.

Theorem 4.7. *If $a_1 < 1, a_2 < 1$, the equilibrium point of the dynamic surge pricing model (4.2) ultimately stabilizes at Cournot equilibrium in the following cases.*

(a) If $\lambda = 0$, i.e. $p_i = p_i(0) - \mu(q_1 + q_2)$ $(i = 1, 2)$, dynamic surge pricing equilibrium degenerates into Cournot equilibrium.

(b) If $\lambda \neq 0$, the equilibrium of dynamic surge pricing model is equivalent to Cournot equilibrium under the following conditions,

$$\begin{cases} p_1(0) = p_2(0) = a, \\ a_2 = a_1 = \frac{3\gamma k}{a} - 1, \\ \mu = \gamma. \end{cases}$$

Theorem 4.7 shows that under certain parameter conditions, the equilibrium of dynamic surge pricing model and that of Cournot model have the same equilibrium price and equilibrium output (sales).

4.3.4 Analysis of key parameters in dynamic surge pricing model

Parameter a_i $(i = 1, 2)$ in the above subsections mainly acts on the rational sale-forecast system (4.1) and indirectly affects the price of the dynamic surge pricing model (4.2). However, the results of model (4.2) are directly regulated by the parameters λ and μ. In what follows, we explore the value ranges of λ and μ and their influence on the price of the dynamic surge pricing model (4.2).

Proposition 4.8. *(a) The price function $p_i(t)$ $(i = 1, 2)$ in dynamic surge pricing model (4.2) increases with respect to λ and decreases about μ.*

(b) The price function $p_i(t)$ of firm i $(i = 1, 2)$ is convex with respect to q_i.

Proposition 4.8 explains that the price function $p_i(t)$ $(i = 1, 2)$ has a maximum value in dynamic evolution, which is affected by parameters λ, μ and q_i $(i = 1, 2)$.

We can obtain that $\mu \in (0, 1)$ from the classical Cournot model. Assuming that the government sets a price ceiling of $\bar{p}$ for a product, we derive the value range of parameter λ about $\bar{p}$ through the following theorem.

Theorem 4.9. *For dynamic surge pricing model* (4.2)*, supposing that* $\mu = \lambda\mu_0$ $(\mu_0 \in R^+)$*, then the value range of* λ *is* $[0, \bar{\lambda}]$*, where* $i, j = 1, 2;\ i \neq j$

$$\begin{cases} \bar{\lambda} = \frac{\bar{p}-p_0}{r_i q_i^m(1-\alpha q_i^m-\alpha a_j q_j^m)-\mu_0(q_i^m+q_j^m)}, \\ q_i^m = \frac{2r_j(r_i-\mu_0)-r_i a_j(r_j-\mu_0)}{4\alpha r_i r_j-\alpha r_i r_j a_i a_j}. \end{cases} \tag{4.4}$$

The price ceiling in Theorem 4.9 represents the legal maximum price in reality, which means that the pricing of enterprises is restricted by the government.

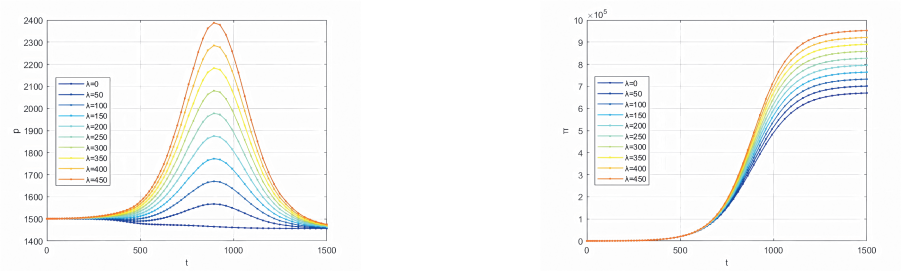
4.4 Numerical simulations

In this paper, a set of numerical experiments is carried out on key parameters to support the theoretical results with parameters that are common in the existing literature (Ren et al., 2020).

4.4.1 Parameter analysis

As shown in Fig. 4.2, we feature the price and profit as a function depending on t for different values of the coefficient λ of marginal sales when other parameters are selected as $a_1 = 0.1$, $a_2 = 0.2$, $r_1 = 0.01$, $r_2 = 0.02$, $\mu = 0.05$, $a = 1500$, $k = 500$, and $c = 0$. When $\lambda = 0$, the

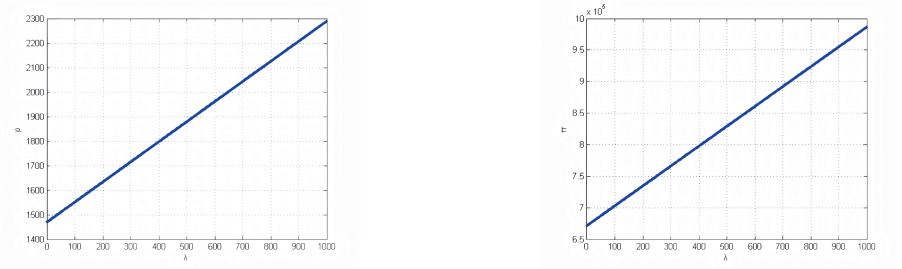
dynamic surge pricing model degrades to the original Cournot model in which the price is only related to total sales. With the increment of λ, the profit and the price increase first and stabilize in the end.



(a) (b)

Figure 4.2: Trajectories for price and profit over time under different values of λ.

Further, Fig. 4.3 shows that price and profit increase as λ increases at a fixed time, which indicates that the dynamic surge pricing model can significantly improve enterprise profit but also lead to a sharp rise in commodity price. Therefore, the government should set a price ceiling as the boundary condition for enterprises to prevent market failure.

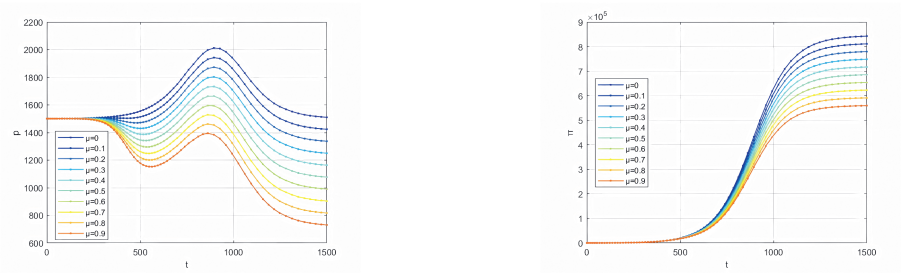


(a) (b)

Figure 4.3: The price and profit of firms versus parameter λ in dynamic surge pricing model.

We fix the parameters $a_1 = 0.1$, $a_2 = 0.2$, $r_1 = 0.01$, $r_2 = 0.02$, $\lambda = 500$,

$a = 1500$, $k = 500$ and $c = 0$. Fig. 4.4 compares the evolution of price and profit with respect to t when the coefficient μ of total sales accepts different values. It can be found that, with the gradual increase of μ, the price and profit of firms decrease at any time. This finding implies that products already on the market can block unsold goods. In the pricing process, the company should control the value of μ at a low level by improving product quality, which means that the price of products is less sensitive to changes in the market.

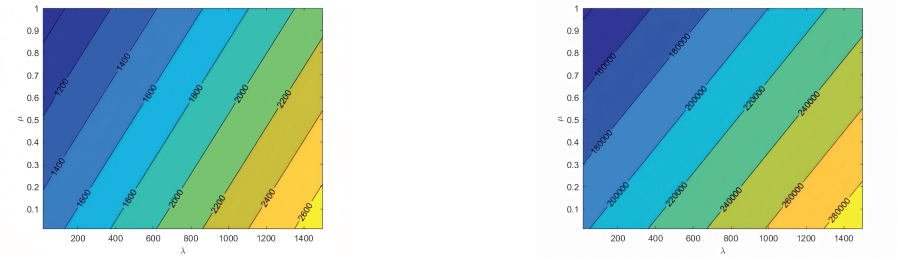


(a) (b)

Figure 4.4: Trajectories for price and profit over time under different values of μ.

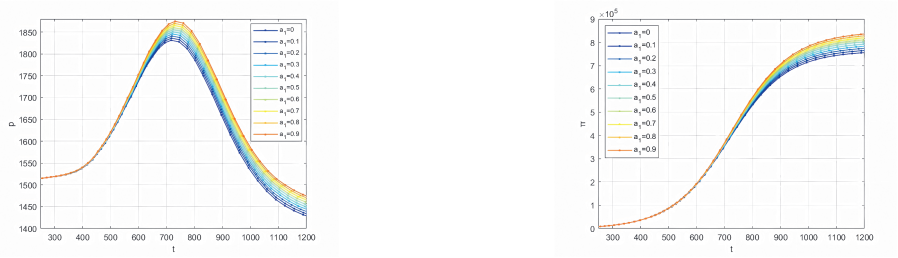
The price and profit are characterized as a function of λ and μ in Fig. 4.5 with parameters $a_1 = 0.1$, $a_2 = 0.2$, $r_1 = 0.01$, $r_2 = 0.02$, $a = 1500$, $k = 500$ and $c = 0$. We can find that the price and profit increase (decrease) with λ (μ) and the change is approximately linear, which verifies the conclusions in Proposition 4.8.

Due to the symmetry of firms, Fig. 4.6 shows the change of price and profit with respect to parameter a_i when $a_j = 0.1$, $r_1 = 0.01$, $r_2 = 0.02$, $\lambda = 400$, $\mu = 0.1$, $a = 1500$, $k = 500$, and $c = 0$. With the increase of a_i, the profit and the price rise first and stabilize in the end. As mentioned above, a_i is the competitiveness of firm i, which can also be described as the inhibition on the sales of competitors' products in the market. As a_i increases, the product of firm i grows faster and reaches a higher equilibrium.



(a) (b)

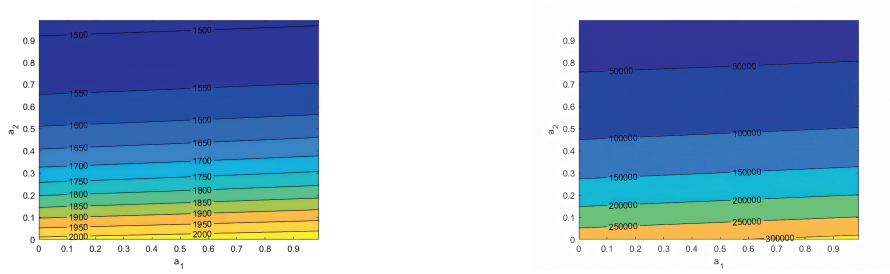
Figure 4.5: The price and profit of firms versus parameters λ and μ in dynamic surge pricing model.



(a) (b)

Figure 4.6: Trajectories for price and profit over time under different values of a_i.

Fig. 4.7 shows the comprehensive effect of a_1 and a_2 on the price and profit of firm 1 with parameters $\lambda = 500$, $\mu = 0.01$ $r_1 = 0.01$, $r_2 = 0.02$, $a = 1500$, $k = 500$ and $c = 0$. We can see that the price and profit of firm 1 increase with a_1 and decrease with a_2. That is, as the potential demand in the market is certain, the sales of each firm are affected by the others. Therefore, firms should compete for market share through advertising, discounts, and other promotional means.

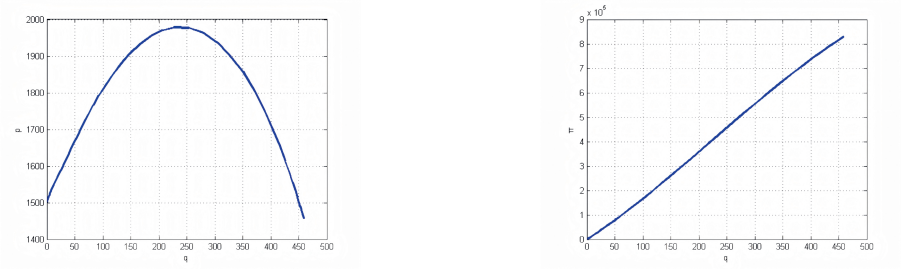


(a) (b)

Figure 4.7: The price and profit of firms versus parameters a_1 and a_2 in dynamic surge pricing model.

By fixing the value parameters $a_1 = 0.1$, $a_2 = 0.2$, $r_1 = 0.01$, $r_2 = 0.02$, $\mu = 0.05$, $\lambda = 500$, $a = 1500$, $k = 500$ and $c = 0$, Fig. 4.8 shows that the price $p_i(t)$ is convex with respect to the sales q_i of firms i $(i = 1, 2)$. It can be concluded that, in the dynamic surge pricing model, the price evolution process has a maximum value, and the boundary of this value is the price ceiling $\bar{p}$. Meanwhile, the profit increases monotonously with respect to sales, that is to say, with each additional unit sold, the profit will increase until the potential total demand is satisfied.

Then we develop a concrete numerical example to compare the profit of the Cournot model with the dynamic surge pricing model under certain parameters.



(a) (b)

Figure 4.8: The price and profit versus sales of firms in the dynamic surge pricing model.

4.4.2 Longitudinal analysis of dynamic surge pricing model

In this subsection, the advantages of the dynamic surge pricing model in a numerical example are demonstrated through longitudinal studies and cross-sectional studies. We first compare the change in profit over time in the dynamic surge pricing model and Cournot model in the long run. Secondly, we intercept the cross-section in the stable state and analyze the profit difference between the two models under different λ to support Theorem 4.7.

Example 4.10. *With the rapid development of the e-commerce market, many traditional off-line sales industries have ventured into online selling. These emerging e-commerce markets often form oligopolies subject to capital controls, such as firms in the mobile phone industry (e.g., Apple and Samsung), ride-hailing platforms (e.g., Uber and Lyft), delivery services (e.g., Meituan and Eleme) and cosmetics manufacturers (e.g., Estee Lauder and Olay). Assuming that the duopoly market sells homogeneous products through online channels, the competition coefficients of the two enterprises are* $a_1 = a_2 = 0.001$. *Considering dynamic surge pricing model* (4.2) *and Cournot model satisfying Theorem* 4.7 *with* $\mu = \gamma = 0.6$, $a = 1500$, $\lambda = 80$, $c = 0$, *the profit differential equation of firm* i $(i = 1, 2)$ *in dynamic surge pricing model*

is as follows,

$$\dot{\pi}_i(t) = (1500 + 80 \cdot \dot{q}_i(t) - 0.6 \cdot (q_1(t) + q_2(t))) \cdot \dot{q}_i(t).$$

The following differential equation describes the profit change of firm i $(i = 1, 2)$ *in Cournot model.*

$$\dot{\pi}_i^c(t) = (1500 - 0.6 \cdot (q_1(t) + q_2(t))) \cdot \dot{q}_i^c(t).$$

In what follows, Fig. 4.9 simulates the long-term profit change mentioned in this example.

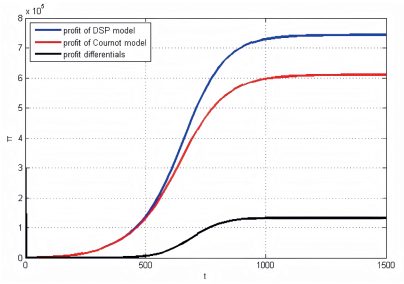


Figure 4.9: The change of profit for firms in the long run.

Through the numerical simulation shown in Fig. 4.9, we obtain that the total profits of the dynamic surge pricing model and Cournot model in equilibrium are 7.43×10^5 and 6.10×10^5, respectively. As the profits of the above two models increase over time, the difference of profits widens until it stabilizes at 1.33×10^5. That is, the profit of the dynamic surge pricing model increases by 21.82% compared with that of Cournot model when the equilibrium is reached under certain conditions according to Theorem 4.7. The numerical result in this example is consistent with the conclusion by Tong et al. (2020), which indicates that compared with general pricing strategies, dynamic surge pricing decisions yield better performance through a large number of data analyses.

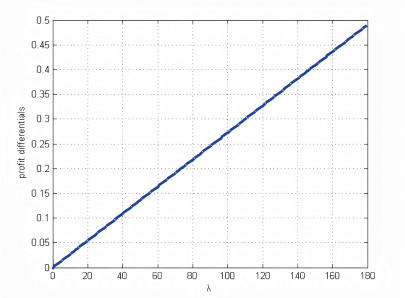


Figure 4.10: The profit difference of firms versus λ between the two model.

Furthermore, Fig. 4.10 describes the change in profit differentials between the dynamic surge pricing model and Cournot model with λ increase in the equilibrium cross-section. According to Fig. 4.10, profit differentials increase with the increase of λ, which is numerically sensitive. Therefore, enterprises and the government should establish corresponding policies to determine the value of λ in practice, which is a problem to be studied in the future.

What's more, we also compare the model's long-term effects on firms and consumers with other literature on dynamic pricing. We collected four kinds of dynamic pricing literature, which are dynamic pricing under inventory change, dynamic pricing under bass forecast, dynamic model of marginal profit adjustment, and data analysis of dynamic pricing. We have listed the differences among them in Table 4.1.

Existing literature (H.-c. Teng and Y.-f. Huang, 2013) has shown that the Lotka-Volterra forecasting model is more effective than the bass model because it is more accurate and requires less data, and we further prove that the dynamic surge pricing model based on the Lotka-Volterra system is more profitable than Cournot model in the long run. Therefore, we can observe that compared with the existing literature, the dynamic surge pricing model not only reduces the dependence of pricing on data but also increases the profitability of enterprises.

Table 4.1: The differences between prior literature and this work.

Typical literature	Research background	Focus	Consumer demand	Long-term results
Lin et al. Lin and Sibdari, 2009	Perishable goods	Inventory changes	Nonlinear demand	Equilibrium
Tong et al. Tong et al., 2020	Two-sided platform	Data analysis	Linear demand	-
Li et al. X. Li et al., 2021	Demand forecast	Bass diffusion model	Linear demand	-
Ren et al. Ren et al., 2020	Output adjustment	Marginal profit adjustment	Linear demand	Stable equilibrium
This paper	Price adjustment	Lotka-Volterra system forecast and marginal sales adjustment	Nonlinear demand	Stable equilibrium

4.5 Conclusion

In this chapter, a dynamic pricing method is proposed in response to the changing potential demand due to efficient product diffusion such as advertising and word of mouth. Based on this research, we provide the following conclusions and recommendations.

- Our results show that the dynamic surge pricing model with dynamic marginal sales adjustment is more profitable compared with Cournot model that only relates to total sales, which suggests the necessity for pricing decisions to take demand trends into account.

- Dynamic surge pricing model suggests that enterprises adjust price in real-time. Enterprises should set a low price to open the market in the early stage of sales, then raise the price to seek greater profit at the demand peak, and cut price in the late stage to reduce loss.

- The limitation of this study mainly lies in the sensitivity of price to parameter λ. That is, the profit of the dynamic surge pricing model increases continuously with the increase of marginal sales, which also reminds the government to regulate the enterprises. The method adopted in this paper is to set a price ceiling to prevent enterprises from vicious competition and price gouging.

4.6 Appendix

Proof of Lemma 4.2. For economical meaning, it is necessary that $q_i \geq 0$ $(i = 1, 2)$, which is equivalent to that $q_i \leq k$ $(i = 1, 2)$. According to system (4.1), let $h_i(t) = \frac{\mathrm{d}q_i(t)}{\mathrm{d}t} = r_i q_i(1 - \frac{q_i}{k} - a_j \frac{q_j}{k})$, then we can calculate that $(i = 1, 2)$

$$\begin{cases} h_i(t) > 0, & \text{if } q_i \in (0, k - a_j q_j), \\ h_i(t) \leq 0, & \text{if } q_i \in (k - a_j q_j, k). \end{cases}$$

By $a_1 < 1$, $a_2 < 1$, it holds that $0 < k - a_j q_j < k,\ i = 1, 2$. Therefore, there exists an inverse function $t = {q_i}^{-1}(q_i)$ of q_i, such that $(i = 1, 2)$

$$\begin{cases} \frac{\mathrm{d}q_i(t)}{\mathrm{d}t} = h_i(t) > 0, & \text{if } t \in (0, {q_i}^{-1}(k - a_j q_j)), \\ \frac{\mathrm{d}q_i(t)}{\mathrm{d}t} = h_i(t) \leq 0, & \text{if } t \in ({q_i}^{-1}(k - a_j q_j), \infty). \end{cases}$$

To sum up, q_i $(i = 1, 2)$ is monotonically increasing first and then non-increasing. Further, we can calculate that the range of sales for each firm is $q_i \in [0, \frac{k - k a_j}{1 - a_i a_j}]$, $i = 1, 2$. □

Proof of Lemma 4.3. For notation simplicity, let $\frac{1}{k} \triangleq \alpha$, then we calculate the second derivative of q_i with respect to time t, which is $(i, j = 1, 2;\ i \neq j)$

$$\begin{aligned} \frac{\mathrm{d}^2 q_i(t)}{\mathrm{d}t^2} &= r_i \dot{q}_i(1 - \alpha q_i - \alpha a_j q_j) + r_i q_i(1 - \alpha \dot{q}_i - \alpha a_j \dot{q}_j) \quad (4.5) \\ &= r_i q_i[\, r_i(1 - \alpha q_i - \alpha a_j q_j)(1 - 2\alpha q_i - \alpha a_j q_j) \\ &- \alpha a_j r_j q_j(1 - a_i \alpha q_i - \alpha q_j)\,]. \end{aligned}$$

Let $g(t) = r_i(1 - \alpha q_i - \alpha a_j q_j)(1 - 2\alpha q_i - \alpha a_j q_j) - \alpha a_j r_j q_j(1 - a_i \alpha q_i - \alpha q_j)$. The convexity of the original equation depends on $g(t)$ because $q(t) \geq 0$. It is easy to know that $g(t)$ is a quadratic function of $q_i(t)$ with a positive quadratic coefficient.

Combining Lemma 4.2, we just have to prove that $g(t)$ has a zero point when $q_i \in [0, \frac{1-a_j}{\alpha(1-a_i a_j)}]$. Then we prove this by the zero point theorem.

First, let $q_i = \frac{1-\alpha a_j q_j}{2\alpha}$, then we can calculate that

$$g(t) = -\alpha r_j a_j q_j (1 - \alpha q_j - \frac{a_i}{2} + \frac{\alpha a_i a_j q_j}{2}). \tag{4.6}$$

Through $q_j < \frac{1-a_i}{\alpha(1-a_i a_j)}$, it holds that $q_j < \frac{2-a_i}{\alpha(2-a_i a_j)}$, and then we can get that $1 - \alpha q_j - \frac{a_i}{2} + \frac{\alpha a_i a_j q_j}{2} > 0$. Therefore, the function $g(t)$ is always negative if $q_i = \frac{1-\alpha a_j q_j}{2\alpha}$.

Second, let $q_i = \frac{1-\alpha a_j q_j}{n\alpha}$ $(n \to \infty)$, then we can calculate that

$$\begin{aligned} g(t) = &\frac{[r_i(n-1)(n-2)\alpha^2 a_j^2 + n\alpha^2 r_j a_j (n - a_i a_j)]q_j^2}{n^2} \\ &- \frac{[2r_i(n-1)(n-2)\alpha^2 a_j + n\alpha r_j a_j (n - a_i)]q_j}{n^2} + r_i. \end{aligned} \tag{4.7}$$

Since differential systems interact and evolve dynamically, we only need to prove that there exists q_j such that $g(t) > 0$.

Let $q_j = 0$, we can obtain that $g(t) = r_i > 0$. What's more, the following case would hold if $q_j = \frac{1}{\alpha}$,

$$g(t) = \frac{r_i(n-1)(n-2)}{n^2}(1 - a_j)^2 + r_j a_j \frac{a_i(1-a_j)}{n}. \tag{4.8}$$

According to L'Hospital's rule, we can get that $g(t) > 0$. Thus, there must exist some range of values of $q_j \in [0, \frac{1-a_i}{\alpha(1-a_i a_j)}]$ such that $g(t) > 0$.

For the function $g(t)$ in the second derivative (4.5), we have shown that $g(t) < 0$ if $q_i = \frac{1-\alpha a_j q_j}{2\alpha}$ and $g(t) > 0$ if $q_i = \frac{1-\alpha a_j q_j}{n\alpha}$ $(n \to \infty)$,

satisfying that $\frac{1-\alpha a_j q_j}{2\alpha} > \frac{1-\alpha a_j q_j}{n\alpha}$. Through the zero point theorem, the function $g(t)$ has a zero point $q_i^0 \in (\frac{1-\alpha a_j q_j}{n\alpha}, \frac{1-\alpha a_j q_j}{2\alpha})$.

From Lemma 4.2 we know that q_i $(i = 1, 2)$ is a monotonically increasing function when $t \in (0, {q_i}^{-1}(k - a_j q_j))$, so there exists a unique $t_2 = {q_i}^{-1}(q_i^0)$ such that the corresponding solution functions $q_i = q_i(t)$ of system (4.1) are convex when $t \in (0, t_2)$ and nonconvex $t \in (t_2, \infty)$, for $i = 1, 2$. □

Proof of Theorem 4.4

Proof. The solution function of system (4.1) is continuous and that $a_1 < 1$ and $a_2 < 1$ are sufficient conditions for both Lemma 4.2 and Lemma 4.3 to be true, so Theorem 4.4 turns out to be true. □

Proof of Theorem 4.6

Proof. From model (4.2) we know that

$$p_i(t) = p_i(0) + \lambda \cdot \dot{q}_i(t) - \mu \cdot (q_1(t) + q_2(t)),\ i = 1, 2.$$

We can observe from the above equation that the stability of p_i $(i = 1, 2)$ is only related to q_i $(i = 1, 2)$. Then the stability analysis of the dynamic surge pricing model is transformed to study whether system (4.1) is stable. Therefore, we first analyze the sufficient conditions for global asymptotic stability of the equilibrium point (q_1^*, q_2^*) in system (4.1).

For economic meaning, it is necessary that $q_1^* > 0$ and $q_2^* > 0$. These conditions are equivalent to

$$\begin{cases} a_1 < 1, \\ a_2 < 1, \\ a_1 a_2 < 1. \end{cases} \quad \text{or} \quad \begin{cases} a_1 > 1, \\ a_2 > 1, \\ a_1 a_2 > 1. \end{cases} \tag{4.9}$$

We define the following positive definite Liapunov function as

$$V(q_1, q_2) = \sum_{i=1}^{2} m_i(q_i - q_i^* - q_i^* \ln \frac{q_i}{q_i^*}), \quad m_i \in R^+.$$

The time derivative of system (4.1) along the solution is

$$\begin{aligned} & \dot{V}(q_1, q_2) \\ = \; & -m_1(q_1 - q_1^*)(\frac{r_1}{k}(q_1 - q_1^*) + \frac{a_2 r_1}{k}(q_2 - q_2^*)) \\ & -m_2(q_2 - q_2^*)(\frac{a_1 r_2}{k}(q_1 - q_1^*) + \frac{r_2}{k}(q_2 - q_2^*)) \\ = \; & -[m_1 \frac{r_1}{k}(q_1 - q_1^*)^2 + m_2 \frac{r_2}{k}(q_2 - q_2^*)^2 \\ & +(m_1 \frac{a_2 r_1}{k} + m_2 \frac{a_1 r_2}{k})(q_1 - q_1^*)(q_2 - q_2^*). \end{aligned} \tag{4.10}$$

The above equation can be regarded as a quadratic form in the variables $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$. $\dot{V}(q_1, q_2)$ is negative definite when $\Delta = (m_1 \frac{a_2 r_1}{k} + m_2 \frac{a_1 r_2}{k})^2 - 4 m_1 m_2 \frac{r_1}{k} \frac{r_2}{k} < 0$. What's more, Δ can also be described as

$$\Delta = (m_1 \frac{a_2 r_1}{k} - m_2 \frac{a_1 r_2}{k})^2 - 4 m_1 m_2 (\frac{r_1}{k} \cdot \frac{r_2}{k} - \frac{a_2 r_1}{k} \cdot \frac{a_1 r_2}{k}) < 0.$$

Let $m_2 = \frac{m_1 a_2 r_1}{a_1 r_2}$, then

$$\Delta = -\frac{4 m_1^2 a_2 r_1}{a_1 r_2} (\frac{r_1}{k} \cdot \frac{r_2}{k} - \frac{a_2 r_1}{k} \cdot \frac{a_1 r_2}{k}) < 0.$$

Compared with conditions (4.9), system (4.1) is globally asymptotically stable at equilibrium point (q_1^*, q_2^*) when $a_1 < 1$, $a_2 < 1$.

Hence, it holds that

$$\begin{cases} \lim\limits_{t\to\infty} q_1(t) = \frac{k(1-a_2)}{1-a_2a_1} = q_1^*, \\ \lim\limits_{t\to\infty} q_2(t) = \frac{k(1-a_1)}{1-a_2a_1} = q_2^*. \end{cases} \tag{4.11}$$

Then we can infer the following equation $(i = 1, 2)$, which is

$$\begin{aligned} &\lim_{t\to\infty} p_i(t) \\ &= \lim_{t\to\infty} (p_i(0) + \lambda\tfrac{dq_i(t)}{dt} - \mu \cdot (q_1(t) + q_2(t)) \\ &= p_i(0) + 0 - \mu\tfrac{k(2-a_2-a_1)}{1-a_2a_1} \\ &= p_i(0) - \mu\tfrac{k(2-a_2-a_1)}{1-a_2a_1} = p_i^*. \end{aligned} \tag{4.12}$$

□

Proof of Theorem 4.7

Proof. Given the equilibrium point (q_i^*, p_i^*) and Cournot point (q_i^{c*}, p_i^{c*}), if they satisfy the equations $(i = 1, 2)$

$$\begin{cases} q_i^* = q_i^{c*}, \\ p_i^* = p_i^{c*}, \end{cases} \tag{4.13}$$

which are equivalent to

$$\begin{cases} \frac{k(1-a_2)}{1-a_2a_1} = \frac{k(1-a_1)}{1-a_2a_1} = \frac{a}{3\gamma}, \\ p_0 - \mu(q_1^* + q_2^*) = a - \gamma(q_1^{**} + q_2^{**}) = \frac{a}{3}. \end{cases} \tag{4.14}$$

The outcome of equation (4.14) can be easily calculated as

$$\begin{cases} p_0 = a, \\ a_2 = a_1 = \frac{3\gamma k}{a} - 1, \\ \mu = \gamma. \end{cases}$$

□

Proof of Proposition 4.8

Proof. Take the partial derivatives of parameters λ and μ of price $(p_i(t))$ in model (4.2), we have

$$\frac{\partial p_i(t)}{\partial \lambda} = r_i q_i (1 - \frac{q_i}{k} - a_j \frac{q_j}{k}) > 0,\ i, j = 1, 2;\ i \neq j, \tag{4.15}$$

and

$$\frac{\partial p_i(t)}{\partial \mu} = -(q_i + q_j) < 0,\ i, j = 1, 2;\ i \neq j. \tag{4.16}$$

According to the above calculation, it follows that the price function $p_i(t)$ $(i = 1, 2)$ is monotonically increasing with respect to λ and monotonically decreasing about μ.

Next, we find the points on $p_i(t)$ $(i = 1, 2)$ where the partial derivative is zero with respect to q_i

$$\frac{\partial p_i(t)}{\partial q_i} = -2\alpha\lambda r_i q_i - \alpha\lambda r_i a_j q_j + \lambda r_i - \mu,\ i, j = 1, 2;\ i \neq j. \tag{4.17}$$

Let $(4.17) = 0$, we can get the solution function q_i^m for maximizing price, that is,

$$q_i^m = \frac{2r_j(r_i - \mu_0) - r_i a_j((r_j - \mu_0))}{4\alpha r_i r_j - \alpha r_i r_j a_i a_j},\ i, j = 1, 2;\ i \neq j.$$

We can calculate that $q_i^m \in (0, k)$, $i = 1, 2$, then the conclusion is drawn that the price function $p_i(t)$ of firm i $(i = 1, 2)$ is concave with respect to q_i. □

Proof of Theorem 4.9

Proof. According to the system (4.1), system (4.2) we know that, for $i, j = 1, 2, i \neq j$,

$$p_i(t) = p_0 + \lambda r_i q_i (1 - \alpha q_i - \alpha a_j q_j) - \mu(q_i + q_j). \tag{4.18}$$

Proposition 4.8 states that p_i is monotonically increasing with respect to λ and concave with respect to q_i. To discuss the maximum of λ, it suffices to find the maximum of p_i with respect to q_i. Without loss of generality, assuming that the relationship of μ and λ is $\mu = \lambda\mu_0$, then we have

$$\begin{cases} \frac{\partial p_i}{\partial q_i} = -2\lambda\alpha r_1 q_1 - \lambda\alpha r_1 a_2 q_2 + \lambda r_1 - \lambda\mu_0 = 0, \\ \frac{\partial p_i}{\partial q_i} = -2\lambda\alpha r_2 q_2 - \lambda\alpha r_2 a_1 q_1 + \lambda r_2 - \lambda\mu_0 = 0. \end{cases} \tag{4.19}$$

After solving equation (4.19), the sale responses can be obtained as,

$$\begin{cases} q_1^m = \frac{2r_2(r_1-\mu_0)-r_1 a_2((r_2-\mu_0))}{4\alpha r_1 r_2 - \alpha r_1 r_2 a_1 a_2}, \\ q_2^m = \frac{2r_1(r_2-\mu_0)-r_2 a_1((r_1-\mu_0))}{4\alpha r_1 r_2 - \alpha r_1 r_2 a_1 a_2}. \end{cases} \tag{4.20}$$

Inserting the above equation in model (4.18), we can get the maximum (p_i^m) of p_i. For any $i, j = 1, 2, i \neq j$,

$$p_i^m = p_0 + \lambda r_i q_i^m (1 - \alpha q_i^m - \alpha a_j q_j^m) - \mu(q_i^m + q_j^m) \leq \bar{p}.$$

The value range of λ can be deduced from the above equation, which is

$$\lambda \leq \frac{\bar{p} - p_0}{r_i q_i^m (1 - \alpha q_i^m - \alpha a_j q_j^m) - \mu_0(q_i^m + q_j^m)} =: \bar{\lambda},\ i, j = 1, 2;\ i \neq j,$$

where q_i^m, $i = 1, 2$ can be obtained in equation (4.20). □

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Summary

Economic development has not only accelerated the dynamics of economic activities, but has also resulted in severe environmental pollution. This thesis applies oligopoly models to analyze sustainable regulation and policy designs. It also uses dynamic models to explore the dynamic evolution of firms' strategy frequencies and prices.

Chapter 2 studies the effect of media supervision on the CSR behaviors of firms through a Stackelberg-Evolutionary game model. Firms play an evolutionary game in which one fraction of firms uses a CSR strategy, while others use a traditional strategy. Without intervention, firms using the CSR strategy will become extinct in the long run. To increase the fraction of firms using the CSR strategy, the media is introduced as a Stackelberg-leader to supervise and expose firms. The media determines the optimal level of supervision, which affects the demand of firms. Firms are Stackelberg-followers and in the evolutionary game under media supervision. The results show that there exists a unique Stackelberg-Evolutionary equilibrium. In equilibrium, there is a strictly positive fraction of firms that use the CSR strategy. This emphasizes the effectiveness of media supervision.

Chapter 3 analyzes the constraints of emission allowances on firms' output and pollution level, and explores the optimal allocation by the government to maximize social welfare. In this chapter, first, the Cournot model is extended by introducing a strategic variable that represents the level of pollution in the profit function of firms. We show that, in equilibrium, each firm adopts the maximum pollution level. To reduce emissions, a Stackelberg game model is proposed to analyze the effect of government intervention. In the first stage, the government determines the allocation of emission allowances to maximize social welfare, balancing economic welfare and social damage. In the second stage, firms play a Cournot game with capacity constraints due

to emission allowances. We show that there is a unique Stackelberg equilibrium. In the general case, in the Stackelberg equilibrium, the government only chooses one out of a limited number of allowance allocations in equilibrium. When the damage function is linear, in most cases, the government-optimal allocation is either to enforce zero pollution or to allow firms to pollute freely.

Chapter 4 develops a dynamic surge pricing model in which sales are forecast by the solution functions of a Lotka–Volterra system. This system determines the pricing model, where price choices by firms are continuously updated. Based on the sales forecast, this chapter extends the inverse demand function by incorporating the impact of marginal sales. The dynamic price is a smooth function that initially increases, then decreases, and ultimately stabilizes at a constant level. Sufficient conditions are derived for the global asymptotic stability of the equilibrium points of the dynamic surge pricing model. Compared with the static Cournot model, the dynamic surge pricing model increases the profitability of firms in the long run.

Samenvatting

Economische ontwikkeling heeft niet alleen de dynamiek van economische activiteiten versneld, maar heeft ook geleid tot ernstige milieuschade. Deze thesis maakt gebruik van oligopoly-model, waaronder de modellen van Stackelberg en Cournot, om duurzame regulering en beleidsontwerp te analyseren. Daarnaast worden dynamische-systemen benadering, zoals de replicator-dynamische vergelijking en Lotka-Volterra-systemen, gebruikt om de dynamische evolutie van strategiefrequenties en prijzen te analyseren.

Het tweede hoofdstuk richt zich op het effect van mediatoezicht op het CSR-gedrag van ondernemingen aan de hand van een Stackelberg-Evolutionary-model. Ondernemingen spelen een evolutionair spel waarin een deel van de ondernemingen een CSR-strategie hanteert, terwijl de overige ondernemingen een traditionele strategie hanteren. Zonder interventie zullen ondernemingen die een CSR-strategie hanteren op de lange termijn uitsterven. Om het aandeel ondernemingen dat een CSR-strategie gebruikt te verhogen, worden de media als leider geïntroduceerd om toezicht te houden op ondernemingen en deze aan de kaak te stellen. Om de nutswaarde te maximaliseren, bepaalt de media het niveau van toezichtinspanningen, die verschillende effecten op de vraag hebben. Ondernemingen zijn volgers en nemen onder mediatoezicht nog steeds deel aan het evolutionaire spel. Uit de resultaten blijkt dat er een uniek Stackelberg-Evolutionary-evenwicht bestaat. In het evenwicht is er een strikt positief aandeel ondernemingen dat de CSR-strategie hanteert. Dit onthult de effectiviteit van mediatoezicht.

Het derde hoofdstuk analyseert de beperkingen die emissierechten opleggen aan de productie en het vervuilingsniveau van ondernemingen en onderzoekt hoe de overheid emissierechten optimaal kan toewijzen om de maatschappelijke welvaart te maximaliseren. Dit

hoofdstuk breidt het Cournot-model uit door een strategievariabele te introduceren die het vervuilingsniveau weergeeft. De resultaten laten zien dat de optimale strategie voor elke onderneming erin bestaat de bovengrens van het vervuilingsniveau in het Cournot-evenwicht te hanteren. Om de uitstoot te verminderen, wordt een Stackelberg-model ontwikkeld om overheidsinterventie te analyseren. In de eerste fase bepaalt de overheid de toewijzing van emissierechten om de maatschappelijke welvaart te maximaliseren, die bestaat uit economisch welzijn en maatschappelijke schade. In de tweede fase spelen ondernemingen een Cournot-model met capaciteitsbeperkingen als gevolg van emissierechten. In het algemene geval zou de overheid in het evenwicht één van een beperkt aantal mogelijke toewijzingen van emissierechten kiezen. Wanneer de schadefunctie lineair is, bestaat de overheid-optimaal toewijzing in de meeste gevallen uit het volledig voorkomen van vervuiling of het vrij laten vervuilen van ondernemingen.

Om de winst gedurende de productlevenscyclus te verhogen, ontwikkelt hoofdstuk 4 een dynamisch surge-pricingmodel, waarin de verkoop wordt voorspeld aan de hand van de oplossingsfuncties van een Lotka-Volterra-systeem. Op basis van de verkoopprognose breidt dit hoofdstuk de inverse vraagfunctie uit door het effect van marginale verkopen erin op te nemen. De dynamische prijs is een gladde functie die aanvankelijk toeneemt, vervolgens afneemt en uiteindelijk stabiliseert op een constant niveau. Er worden voldoende voorwaarden afgeleid voor de globale asymptotische stabiliteit van de evenwichtspunten van het dynamische surge-pricingmodel. In vergelijking met het Cournot-model verhoogt het dynamische surge-pricingmodel de winstgevendheid van ondernemingen op de lange termijn.

Impact of the thesis

Rapid economic development has exacerbated environmental problems, making them one of the biggest threats facing humanity today. To address this challenge, the United Nations calls for urgent action to combat climate change and its impacts (SDG 13). As firms' production activities are a major source of environmental pollution, it is essential to encourage them to adopt more sustainable production practices. However, firms may face a conflict between sustainable production and profitability. Regulatory intervention in firms' strategic choices is crucial to incentivizing sustainable production. Focusing on sustainable regulation and pricing strategies, this thesis discusses the scientific and social impacts of each chapter.

Chapter 2 aims to analyze the effect of media supervision on the sustainable production of firms. We propose a novel modeling approach that nests the evolutionary game into the Stackelberg game, and find that media supervision can ensure that the firms' strategies align with social responsibilities. This provides a theoretical basis for practical cases, with particular attention to the media's profit modes, cost structures, and influence on firms' demand. For policymakers, we emphasize the necessity of a combination of media supervision and environmental taxes for sustainable development.

To reduce emissions, Chapter 3 explores the government-optimal allocation of emission allowances to maximize social welfare. We propose a Stackelberg game model in which the government moves first to allocate emission allowances, and then firms play a Cournot game with capacity constraints due to emission allowances. When the damage function is linear, in most cases, the government-optimal allocation is either to enforce zero pollution or to allow firms to pollute freely. Thus, the standard procedures used in practice to allocate allowances, such as auctions and grandfathering, may be suboptimal for maximizing

social welfare. The policy behind such methods of allowance allocation may represent the government's trade-off between environmental protection and economic development.

Chapter 4 develops a dynamic surge pricing model in which sales are forecast by the solution functions of a Lotka-Volterra system. Compared with the static Cournot model, the dynamic surge pricing model increases the profitability of firms in the long run. The dynamic surge pricing model suggests that enterprises adjust prices in real-time. Specifically, firms should set a low price to open the market in the early stage of sales, then raise the price to seek greater profit at the demand peak, and cut the price in the late stage to reduce loss. We also suggest to the government that a price ceiling can be set to prevent enterprises from vicious competition and price gouging.

The results of this thesis promote the integration and development of policy design, sustainable production, and dynamic economic activities.

About the author

Yanru Sun was born in Xi'an, P.R. of China. In 2015, she became a student at Northwestern Polytechnical University and received a Bachelor of Science degree in 2019. After graduation, she continued her studies for a master's degree at Northwestern Polytechnical University, supervised by Prof. Dr. Hao Sun. One year later, she decided to join the Master-PhD project and subsequently became a PhD student under Prof. Dr. Xueliang Li in 2021. From September 2024, she was awarded a scholarship under the State Scholarship Fund to pursue study in the School of Business and Economics, Maastricht University, as a joint PhD student, under the supervision of Prof. Dr. Dries Vermeulen and Dr. Christopher Kops.


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